The Queen's Gambit: Explaining the Superstar Effect Using Evidence from Chess [*]

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Abstract. Superstars exist in classrooms and workplaces. Superstars can either intimidate others by making their peers put in less effort, or they can encourage others by inspiring everybody to "step up" their game. In this study, we analyze direct and indirect effects of a superstar on their peers using evidence from chess. We find that the direct superstar effect is always negative. The indirect superstar effect depends on the intensity of the superstar: if the skill gap between the superstar and the rest is small (large), there is a positive (negative) peer effect.

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1. Introduction

"When you play against Bobby [Fischer], it is not a question of whether you win or lose. It is a question of whether you survive."

–Boris Spassky, World Chess Champion, 1969 - 1972.

Maximizing their employees' efforts is one of the chief goals of the firm. To this extent, firms typically encourage competition among their employees and allocate bonuses according to their performance and effort. At the same time, firms want to hire the best workers – preferably, the ones who are "superstars" in their fields. For this reason, it is not unusual to see million-dollar hiring contracts among the Forbes top 500 firms.

However, hiring a superstar employee might potentially cause unintentional side effects. Brown (2011) took a creative approach to analyze these potential side effects by considering a famous golf superstar: Tiger Woods. Her goal was to uncover whether Tiger Woods had a positive or negative effect on his competitors' performance. She compared performances in tournaments with and without Tiger Woods and unveiled that there was a sharp decline in performance in tournaments where Tiger Woods competed. This evidence points out that Tiger Woods, as a superstar, creates a psychological pressure on his competitors which has a discouraging effect, causing them to perform worse than their typical performance.

In this paper, we analyze the superstar effect using chess data.[1] Chess provides a clean setting to analyze the superstar effect for the following reasons: First, non-player related factors are minimal to non-existent in chess since every chess board is the same for all players.[2] Second, the move-level performance indicators can be obtained with the use of computer algorithms that can evaluate the quality of each move and estimate the complexity of each unique position. Third, multiple chess superstars exist who lived in different time periods and come from different backgrounds, enhancing

[1]There is growing literature studying a broad range of questions using data from chess competitions. For example, Levitt et al. (2011) test whether chess masters are better at making backward induction decisions. Gerdes and Gränsmark (2010) test for gender differences in risk-taking using evidence from chess games played between male and female players, where they find that women choose more risk-averse strategies playing against men. On the one hand, Backus et al. (2016) and Smerdon et al. (2020) find that female players make more mistakes playing against male opponents with similar strength. On the other hand, Stafford (2018) has an opposite finding that women perform better against men with similar Elo ratings. Dreber et al. (2013) test the relationship between attractiveness and risk-taking using chess games.

[2]There is no compelling reason to expect a systematic difference in the environmental factors to directly affect a tournament performance. However, Künn et al. (2019) and Klingen and van Ommeren (2020) find that indoor air quality has effects on performance and risk-taking behavior of chess players.

the external validity of the study.[3]

To begin with, we present a two-player contest model with a "superstar." Our theory suggests that the skill gap between the superstar and the other player is crucial to determine the superstar effect on the competition. If this gap is small, then the superstar effect is positive: both players exert high effort. However, when the skill gap is large, the superstar effect is negative: both players lower their efforts. Our theory provides explanations for different superstar effects in the literature. The negative superstar effect in golf is found not only in Brown (2011), but also in Tanaka and Ishino (2012)[4], while the positive superstar effect in track and field events is found in Hill (2014). He compares the performance of athletes in runs where Usain Bolt is competing and where Usain Bolt is not present, finding that athletes perform much better when Usain Bolt is competing. This can be attributed to non-superstar athletes being motivated by having Usain Bolt running just "within their reach", enabling them to push one step further and show extra effort.

Then, we test our theory on five different male and female chess superstars who come from different backgrounds and time periods: Magnus Carlsen, Garry Kasparov, Anatoly Karpov, Bobby Fischer, and Hou Yifan.[5] We are looking for direct (individual competition with a superstar) and indirect (performance in a tournament with a superstar) superstar effects in chess tournaments. To find these effects, we analyze 2.1 million move-level observations from elite chess tournaments that took place between 1962 to 2019 with the world's top chess players. Our main performance indicator is unique to chess: the "Average Centipawn Loss" (ACPL), which shows the amount of error a player commits in a game.[6] In chess, a player's goal is to find the optimal move(s). Failing to do so would result in mistake(s), which the ACPL metric captures. Having multiple mistakes committed in a game almost certainly means losing at the top level chess tournaments. We then test the following hypotheses:

1. **Direct effect:** Do players commit more mistakes (than they are expected to) playing head-to-head against a superstar?

2. **Indirect effect:** Do players commit more mistakes (than they are expected to) in games played against each other *if* a superstar is present in the tournament as a competitor?

[3] In the media, "The Queen's Gambit" gives a realistic portrayal of a chess superstar. The protagonist, Beth Harmon, is a superstar who dominates her peers in tournaments. In this paper, we analyze the real-life chess superstar effect on their peers in actual tournaments.

[4] Their superstar is Masashi Ozaki who competed in the Japan Golf Tour and dominated the tournaments he participated in throughout the 1990s.

[5] We discuss why these players are "chess superstars" in Section 3.

[6] The Average Centipawn Loss is also referred to as "mean-error". We provide details on how we use this metric in Section 3.4.

Holding everything else constant, a player should be able to show the same performance in finding the best moves in two "similarly complex" chess positions. The difficulty of finding the optimal moves – assuming players show full effort – is related to two main factors: (1) External factors impacting a player. For instance, being under pressure can lead the player to choke, resulting in more mistakes. (2) The complexity of the position that the player faces. If both players are willing to take risks, they can opt to keep more complex positions on the board, which raises the likelihood that a player commits a mistake. To isolate the "choking effect", we construct a complexity metric using a state-of-the art Artificial Neural Network (ANN) algorithm that is trained with an independent sample with more than 2 million moves.[7] By controlling board complexity, we compare games with identical difficulty levels. If a player commits more mistakes against the superstar (or in the presence of a superstar) in similarly complex games, it must be that either (i) the player chokes under pressure (that is, even if the player shows full effort, the mental pressure of competing against a superstar results in under-performance) or (ii) the player gives up and does not show full effort, considering his or her ex-ante low winning chances (this results in lower performance with more mistakes committed), or both (i) and (ii).

We find a strong **direct superstar effect:** in similarly complex games, players commit more mistakes and perform below their expected level when they compete head-to-head against the superstar. This result can be explained by both the choking and the giving up effects. Consequently, players are less likely to win and more likely to lose in these games compared to their games against other opponents.

We show that the **indirect superstar effect** depends on the skill gap between the superstar and the competition. As our theory predicts, we find that if this gap is small, the indirect superstar effect is positive: it seems that the players believe they indeed have a chance to win the tournament and exert higher effort. The data shows that the top 25 percent of the tournament participants improve their performances and commit fewer mistakes. However, if the skill gap is large, then the indirect superstar effect is negative: it seems that players believe that their chances to win the tournament are slim, and/or that competing at the same tournament with a superstar creates psychological pressure. As a result, the top players show an under-performance with more mistakes and more losses. Interestingly, there is a tendency for the top players to play more complex games in tournaments with a superstar. This suggests that the choking effect is more dominant than the giving up effect.

[7] The details of the algorithm are provided in Section 3.5.

Our results provide clear takeaways for organizations: hiring a superstar can have potential spillover effects, which could be positive for the whole organization if the superstar is slightly better than the rest of the group. However, the organization can experience negative spillover effects if the skill gap between the superstar and the rest of the group is substantial. Thus, managers should compare the marginal benefit of hiring an "extreme" superstar to the potential spillover costs on the whole organization.[8] Moreover, hiring a marginally-better superstar can act as a performance inducer for the rest of the team.

The superstar literature started from Rosen (1981), who makes the first contribution in the understanding of "superstars" by pointing out how skills in certain markets become excessively valuable. One of the most recent theoretical contributions in the "superstar" literature is Xiao (2020), who demonstrates the possibility of having positive or negative incentive effects when a superstar participates in a tournament. These effects depend on the prize structure and the participants' abilities.

Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Moldovanu and Sela (2001) describe how to design optimal contracts in rank-order tournaments. Prendergast (1999) provides a review on incentives in workplaces.

The empirical sports superstar literature started from Brown (2011)[9] and is ranging from professional track and field competitions to swimming. Yamane and Hayashi (2015) compare the performance of swimmers who compete in adjacent lanes and find that the performance of a swimmer is positively affected by the performance of the swimmer in the adjacent lane. In addition, this effect is amplified by the observability of the competitor's performance. Specifically, in backstroke competitions where observability of the adjacent lane is minimal, there appears to be no effect, whereas the effect exists in freestyle competitions with higher observability. Jane (2015) uses swimming competitions data in Taiwan and finds that having faster swimmers in a competition increases the overall performance of all the competitors participating in the competition.

Topcoder and Kaggle are the two largest crowdsourcing platforms where contest organizers can run online contests offering prizes to contestants who score the best in finding a solution to a diffi-

[8] Mitigating the negative effects by avoiding within-organization pay-for-performance compensation schemes is a possibility. However, it is challenging to eliminate all competition in an organization.

[9] Connolly and Rendleman (2014) and Babington et al. (2020) point out that an adverse superstar effect may not be as strong as suggested by Brown (2011). They claim that this result is not robust to alternative specifications and suggest that the effect could work in the opposite direction – that the top competitors can perhaps bring forth the best in other players' performance. In addition, Babington et al. (2020) provide further evidence using observations from men's and women's FIS World Cup Alpine Skiing competitions and find little to no peer effects when skiing superstars Hermann Maier and Lindsey Vonn participate in a tournament. Our theory can suggest an explanation for these findings.

cult technical problem stated at the beginning of the contest. Archak (2010) finds that players avoid competing against superstars in Topcoder competitions. Studying the effect of increased competition on responses from the competitors, Boudreau et al. (2016) discover that lower-ability competitors respond negatively to competition, while higher-ability players respond positively. Zhang, Shunyuan and Singh, Param Vir and Ghose, Anindya (2019) suggest that there may potentially be future benefits from competitions with superstars: the competitors will learn from the superstar. This finding is similar to the positive peer effects in the workplace and in the classroom, see Mas and Moretti (2009), Duflo et al. (2011), Cornelissen et al. (2017), Moreira (2019).

The rest of the paper is organized as follows: Section 2 presents a two-player tournament model with a superstar. Section 3 gives background information on chess and describes how chess data is collected and analyzed. Section 4 provides the empirical design. Section 5 presents the results, and section 6 concludes.

2. Theory

In this section, we consider a two-player contest in which player 1 competes against a superstar, player 2.[10] Player 1 maximizes his expected payoff, consisting of expected benefits minus costly effort[11]

$$\max_{e_1} \quad \frac{e_1}{(e_1+\theta e_2)} V_1 - e_1,$$

where e_i is the effort of player $i = 1, 2$, V_1 is a (monetary or rating/ranking) prize which player 1 can win, and θ is the ability of player 2. We normalize the ability of player 1 at one. Player 2, a superstar, has high ability $\theta \geq 1$ and maximizes her expected payoff:

$$\max_{e_2} \quad \frac{\theta e_2}{(e_1+\theta e_2)} V_2 - e_2,$$

where V_2 is the prize that player 2 can win. Note that θ is not only the ability of player 2, but also the ratio of that player's abilities.[12]

[10] Tullock (1980) discussed a similar model, but did not provide a formal analysis.

[11] We assume that costs are linear functions.

[12] For chess professionals, prizes are monetary payments as well as Elo rating points (which we discuss in section 3.1) at the conclusion of a tournament. High Elo rating increases the probability to be invited to the future elite chess tournaments and is even more important than a monetary reward.

The first order conditions for players 1 and 2 are

$$\frac{\theta e_2}{(e_1+\theta e_2)^2}V_1 - 1 = 0,$$

and

$$\frac{\theta e_1}{(e_1+\theta e_2)^2}V_2 - 1 = 0.$$

Therefore, in an equilibrium

$$\frac{e_2}{e_1} = \frac{V_2}{V_1}.$$

We can state our theoretical results now.

Proposition 1 *Suppose that $V_1 > V_2$. Then, there exists a unique equilibrium in the two-player superstar contest model, where player $i = 1,2$ exerts effort*

$$e_i^* = \frac{\theta V_1 V_2}{(V_1+\theta V_2)^2} V_i.$$

In the equilibrium, player $i = 1,2$ wins the contest with the probability p_i^*, where

$$(p_1^*, p_2^*) = \left(\frac{V_1}{V_1+\theta V_2}, \frac{\theta V_2}{V_1+\theta V_2}\right).$$

We assume that the prize for the underdog is greater than the prize for the superstar in the two-player contest: everyone expects the superstar to win the competition and her victory is neither surprising, nor too rewarding. However, the underdog's victory makes him special, which is also evident from rating point calculations in chess: a lower rated player gains more rating points if he wins against a higher ranked player.[13] It follows from proposition 1 that the underdog, player 1, always exerts higher effort than the superstar, player 2, in the equilibrium, since $V_1 > V_2$. In addition, underdog's winning chances decrease in the superstar abilities. We have the following comparative statics results.

Proposition 2 *Suppose that $V_1 > V_2$. Then, individual equilibrium efforts increase in the superstar ability if $\theta^* < \frac{V_1}{V_2}$ and decrease if $\theta^* > \frac{V_1}{V_2}$. Individual equilibrium efforts are maximized if the superstar ability is $\theta^* = \frac{V_1}{V_2}$.*

[13]The statement of Proposition 1 holds without the assumption about prizes.

Proposition 2 gives a unique value of the superstar ability which maximizes individual and total equilibrium efforts. This observation suggests the best ability of a superstar for the contest designer.

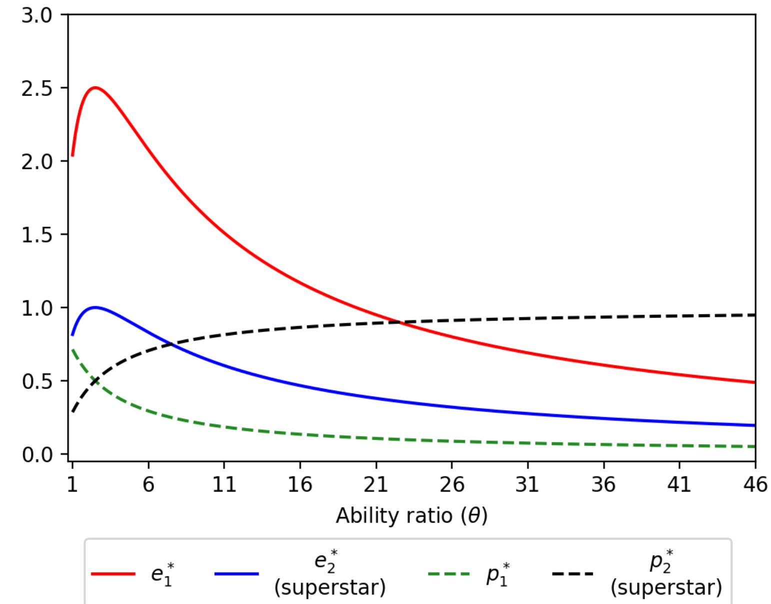

Figure 1: Equilibrium effort and winning probabilities with prizes $V_1 = 10$ and $V_2 = 4$.

Figure 1 illustrates this proposition and shows how equilibrium efforts and winning probabilities change for different levels of superstar abilities if $V_1 = 10$ and $V_2 = 4$. When the ability ratio is small, effort levels for both players increase. As the ability ratio increases, both players decrease their efforts. In other words, if the gap between the superstar and the underdog abilities is small, the superstar effect is positive as both players exert higher efforts. However, if the superstar is much better than the underdog, then both players shirk in their efforts and the superstar effect is negative.

3. Data

3.1 Chess: Background

"It is an entire world of just 64 squares."

–Beth Harmon, The Queen's Gambit,
Netflix Mini-Series (2020)

Chess is a two-player game with origins dating back to 6th century AD. Chess is played over a 8x8 board with 16 pieces for each side (8 pawns, 2 knights, 2 bishops, 2 rooks, 1 queen, and 1 king). Figure 2 shows a chess board. Players make moves in turns, and the player with the white pieces moves first. The ultimate goal of the game is to capture the enemy king. A game can end in three ways: white player wins, black player wins, or the game ends in a draw.

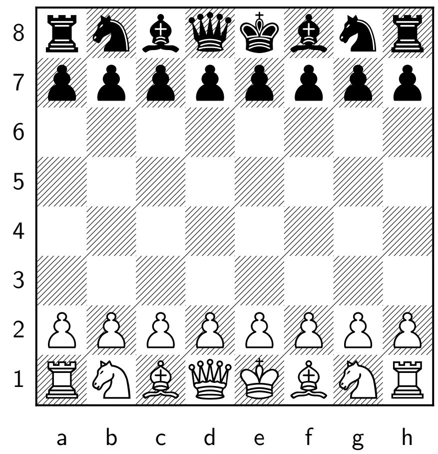

Figure 2: A chess board

The possible combinations of moves in a chess game is estimated to be more than the number of atoms in the universe.[14] However, some moves are better than others. With years of vigorous training, professional chess players learn how to find the best moves by employing backward-induction and calculating consequences of moves to a certain complexity level. Failing to find the best move(s) in a position would result in a "blunder" or a "mistake" which typically leads to the player losing their game at the top level if a player commits multiple blunders or mistakes. The player who performs better overall is the player who manages to find the correct moves more often.

The standard measure of player strength in chess is the Elo rating system first adopted by FIDE in 1970. This system was created by the Hungarian physicist Arpad Elo (Elo 1978). Elo considers the performance of a player in a given game as a random variable normally distributed around her unobservable true ability. Each player gets a starting Elo rating which is updated according to the outcome of each game via

$$ELO_{R,t+1} = ELO_{R,t} + K\left[S_i - E_t\left(S_i \mid R_i, R_j\right)\right], \tag{1}$$

where S_i is the outcome of a game such that $S_i = 1$ if player i wins the game, $S_i = 0$ if player i loses the game, and $S_i = 1/2$ if the game ended in a draw. $E_t\left(S_i \mid R_i, R_j\right)$ is the expected probability of player i winning the game given the Elo ratings of the two players R_i and R_j which equals

[14] A lower bound on the number of possible moves is 10^{120} moves, per Shannon (1950) while the number of atoms in the observable universe is estimated to be roughly 10^{80}.

$E\left(S_i \mid R_i, R_j\right) = \Phi\left(\frac{R_i - R_j}{400}\right)$ where $\Phi(.)$ is the c.d.f. of the normal distribution.[15] K is a parameter for rate of adjustment.

This rating system allows comparisons of players' strengths. For instance, every month, FIDE publishes Elo ratings of all chess players. The Top 10 players are considered the most elite players in the world who earn significant amounts of prizes and sponsorships. Moreover, chess titles have specific Elo rating requirements. For instance, the highest title in chess, Grandmaster, requires the player to have an Elo rating 2500 or higher.[16]

Over the past decades, computer scientists have developed algorithms, or "chess engines" that exploit the game-tree structure of chess. These engines analyze each possible tree branch to come up with the best moves. The early chess engines were inferior to humans. After a few decades, however, one chess engine developed by IBM in the 1990s, Deep Blue, famously defeated the world chess champion at the time, Garry Kasparov, in 1997. This was the first time a world chess champion lost to a chess engine under tournament conditions. Since then, chess engines have passed well beyond the human skills. As of 2021, Stockfish 11 is the strongest chess engine with an Elo rating of 3497.[17] In comparison, the current world chess champion, Magnus Carlsen, has an Elo rating of 2862.[18]

In addition to finding the best moves in a given position, a chess engine can be used to analyze the games played between human players.[19] The quality of a move can be measured numerically by evaluating the move chosen by a player and comparing it to the list of moves suggested by the chess engine. If the move played by a player is considered a bad move by the engine, then that move is assigned a negative value with its magnitude depending on the engine's evaluation.[20]

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[15] The probability that player i wins a game against player j is a function of their true abilities. Let p_i be the performance of player i, with $p_i \sim N(\mu_i, \sigma^2)$ and p_i $i = 1,,..n$ independent draws. Player i wins if $P(p_i > p_j)$ or $P(p_i - p_j > 0)$. With p_i and p_j independent, $p_i - p_j \sim N(\mu_i - \mu_j, 2\sigma^2)$. Therefore $P(p_i - p_j > 0) = P\left(\frac{(p_i - p_j) - (\mu_i - \mu_j)}{\sqrt{2\sigma^2}} > \frac{-(\mu_i - \mu_j)}{\sqrt{2\sigma^2}}\right) = P\left(z > \frac{-(\mu_i - \mu_j)}{\sqrt{2\sigma^2}}\right) = P\left(z < \frac{(\mu_i - \mu_j)}{\sqrt{2\sigma^2}}\right) = \Phi\left(\frac{\mu_i - \mu_j}{\sqrt{2\sigma^2}}\right)$ where z is the standard normal.

[16] Our sample consists of the very elite chess players, often called "Super GMs", with Elo ratings higher than 2700 in most cases.

[17] Modern chess engines, such as Stockfish, have much higher Elo ratings compared to humans. Most modern computers are strong enough to run Stockfish for analyzing chess positions and finding the best moves, which is the engine we use in our analyses.

[18] The highest Elo rating ever achieved by a human was 2882 in May 2014 by Magnus Carlsen.

[19] Every chess game played at the top level is recorded, including all the moves played by the players.

[20] Engine evaluation scores in chess have no impact on the game outcomes. Engines are used in post-game analysis for learning and research purposes. They are also used during live broadcasting, such that the audience can see which player maintains an advantage. The use of a computer engine by a player during a game is against fair play rules and is equivalent to using Performance Enhancing Drugs (PEDs) for other sports.

3.2 Chess Superstars

The first official world chess champion is Wilhelm Steinitz who won the title in 1886. Since Steinitz, there have been sixteen world chess champions in total. Among these sixteen players, four have shown an extraordinary dominance over their peers: Magnus Carlsen, Garry Kasparov, Anatoly Karpov, and Bobby Fischer.[21] We present evidence why these players were so dominant and considered "superstars" in their eras. Specifically, we define a superstar as a player who satisfies the following conditions: (i) be the world chess champion; (ii) win at least 50% of all tournaments participated in;[22] (iii) maintain an Elo rating at least 50 points above the average Elo rating of the world's top 10 players (this condition must hold for the post-1970 era when Elo rating was introduced); (iv) have such a high Elo rating that just winning an elite tournament is not sufficient to gain Elo rating points. We define an elite tournament, a tournament which has (1) at least two players from the world's Top 10 and (2) the average Elo rating in the tournament is within 50 points of the average Elo rating in tournaments with a superstar.

Magnus Carlsen is the current world chess champion, who first became champion in 2013 at age 22. He reached the highest Elo rating ever achieved in history. Garry Kasparov was the world champion from 1985-2000 and was the number one ranked chess player for 255 months, setting a record for maintaining the number one position for the longest duration of time. For comparison, Tiger Woods was the number one ranked player in the world for a total of 683 weeks, the longest ever in golf history. Anatoly Karpov was the world champion before Kasparov in the years 1975-1985. He won over 160 tournaments, which is a record for the highest number of tournaments won by a chess player.[23] Bobby Fischer was the world champion before Karpov between 1972 - 1975, winning all U.S. championships he played in from 1957 (at age 14) to 1966. Fischer won the 1963

[21] In his classic series, "My Great Predecessors", Kasparov (2003) gives in-depth explanations about his predecessors, outlining qualities of each world champion before him. In this paper, we consider the "greatest of the greatest" world champions as "superstars" in their eras.

[22] For comparison, Tiger Woods won 24.2 percent of his PGA Tour events.

[23] Kasparov (2003) shares an observation on Karpov's effect on other players during a game in Moscow in 1974: *"Tal, who arrived in the auditorium at this moment, gives an interesting account: "The first thing that struck me (I had not yet seen the position) was this: with measured steps Karpov was calmly walking from one end of the stage to the other. His opponent was sitting with his head in his hands, and simply physically it was felt that he was in trouble. 'Everything would appear to be clear,' I thought to myself, 'things are difficult for Polugayevsky.' But the demonstration board showed just the opposite! White was a clear exchange to the good – about such positions it is customary to say that the rest is a matter of technique. Who knows, perhaps Karpov's confidence, his habit of retaining composure in the most desperate situations, was transmitted to his opponent and made Polugayevsky excessively nervous."* p. 239 "My Great Predecessors" Vol 5.

U.S. chess championship with a perfect 11 out of 11 score, a feat no other player has ever achieved.[24]

In addition to the four male superstars, we consider a female chess superstar: Hou Yifan, a four time women's world chess champion between the years 2010-2017. She played three women's world chess championship matches in this period and did not lose a single game against her opponents, dominating the tournaments from 2014 until she decided to stop playing in 2017.[25]

Figures A.2–A.6 show how the four world chess champions: Carlsen, Kasparov, Karpov and Hou Yifan performed compared to their peers across years.[26] The Elo rating difference between each superstar and the average of world's top 10 players in each corresponding era is about 100 points. This rating gap is very significant, especially at top-level competitive chess. For instance, the expected win probabilities between two players with a gap of 100 Elo rating points are approximately 64%-36%.

Figures A.13–A.17 show individual tournament performances across years for each superstar with the vertical axis showing whether the superstar gained or lost rating points at the end of a tournament. For instance in 2001, Kasparov played in four tournaments and won all of them. In one of these tournaments, he even lost rating points despite winning. For the world's strongest player, winning a tournament is not sufficient to maintain or gain rating points because he also has to win decisively.[27]

Table 1 presents statistics of the superstars' dominance. Panels A-E include the World's Top 10 chess players for the corresponding era and a summary of their tournament performances. For example, Magnus Carlsen participated in 35 tournaments with classical time controls between 2013 and 2019, winning 21 of them. This 60% tournament win rate is two times higher than World's #2 chess player, Fabiano Caruana, who has a tournament win rate of 30%. A more extreme case is Anatoly Karpov, who won 26 out of 32 tournaments, which converts to an 81% tournament win rate while the runner up Jan Timman had a tournament win rate of 22%.[28]

[24] Kasparov (2003) on Fischer's performance in 1963 U.S. championship: *"Bobby crushed everyone in turn, Reshevsky, Steinmeyer, Addison, Weinstein, Donald Byrne... Could no one really withstand him?! In an interview Evans merely spread his hands: 'Fantastic, unbelievable...' Fischer created around himself such an energy field, such an atmosphere of tension, a colossal psychological intensity, that this affected everyone."* See p. 310 "My Great Predecessors" Vol 4.

[25] Not losing a single game in world championship matches is a very rare phenomenon since the world champion and the contestant are expected to be at similar levels.

[26] Elo rating information is not available for Fischer's era. FIDE adopted the Elo rating system in 1970.

[27] See Figures A.13–A.17 for cases in which the superstar player won a tournament, but nevertheless lost rating points. The superstar must win a tournament by a large margin to maintain #1 rating level.

[28] Restricting the runner-ups' tournament wins to tournaments in which a superstar participated lowers their tournament win rate significantly. Tables are available upon request.

Table 1: World's Top 10 chess players and their tournament performances

years: 2013-2019 *PANEL A*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Carlsen, Magnus	21	35	60%	2855	0.352	0.576	0.072	14.413	26.802	16,104	303
Caruana, Fabiano	15	49	30%	2802	0.283	0.592	0.126	16.758	28.280	22,205	447
So, Wesley	6	28	21%	2777	0.226	0.666	0.108	14.928	25.717	11,622	263
Aronian, Levon	5	33	15%	2788	0.196	0.662	0.142	16.059	25.654	13,455	294
Mamedyarov, Shakhriyar	4	24	16%	2777	0.172	0.674	0.154	15.050	26.339	9,405	216
Giri, Anish	3	32	9%	2770	0.149	0.719	0.131	14.873	26.202	14,224	304
Karjakin, Sergey	3	29	10%	2768	0.168	0.689	0.143	15.947	26.938	12,764	281
Nakamura, Hikaru	3	35	8%	2779	0.218	0.622	0.160	15.823	27.398	15,349	327
Vachier Lagrave, Maxime	3	27	11%	2777	0.163	0.703	0.134	14.539	26.842	10,227	232
Grischuk, Alexander	0	16	0%	2777	0.183	0.633	0.184	18.081	27.539	6,852	146

years: 1995-2001 *PANEL B*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Kasparov, Garry	17	22	77%	2816	0.439	0.510	0.051	17.595	28.509	18,082	488
Kramnik, Vladimir	12	30	40%	2760	0.322	0.621	0.058	16.743	26.071	22,782	610
Anand, Viswanathan	9	25	36%	2762	0.305	0.595	0.099	19.337	28.328	18,879	517
Topalov, Veselin	6	26	23%	2708	0.279	0.514	0.207	21.193	22,081	28.801	515
Ivanchuk, Vassily	4	17	23%	2727	0.255	0.582	0.164	19.626	27.045	13,673	362
Adams, Michael	3	22	13%	2693	0.255	0.575	0.169	19.096	27.382	17,472	421
Short, Nigel D	3	18	16%	2673	0.272	0.475	0.253	22.717	29.080	13,538	348
Svidler, Peter	3	13	23%	2684	0.234	0.599	0.167	19.340	27.573	9,458	260
Karpov, Anatoly	2	12	16%	2742	0.214	0.679	0.107	18.292	26.457	8,966	211
Shirov, Alexei	2	26	7%	2706	0.288	0.460	0.253	21.865	29.014	22,277	529

Table 1 (cont): World's Top 10 chess players and their tournament performances

years: 1976-1983 *PANEL C*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Karpov, Anatoly	26	32	81%	2707	0.432	0.524	0.044	17.429	25.244	14,581	391
Timman, Jan H	7	32	21%	2607	0.333	0.525	0.142	20.777	26.074	17,690	305
Larsen, Bent	5	27	18%	2607	0.383	0.342	0.274	23.726	27.251	17,495	193
Kasparov, Garry	3	5	60%	2652	0.429	0.487	0.084	20.459	25.634	2,207	53
Portisch, Lajos	3	20	15%	2634	0.324	0.516	0.160	20.471	25.729	10,744	204
Tal, Mihail	3	11	27%	2632	0.271	0.652	0.077	19.898	24.915	5,505	131
Petrosian, Tigran V	2	12	16%	2608	0.244	0.652	0.103	20.810	23.089	5,257	100
Spassky, Boris Vasilievich	2	16	12%	2624	0.196	0.697	0.107	19.789	24.771	6,115	157
Beliavsky, Alexander G	1	5	20%	2596	0.320	0.457	0.223	23.923	28.210	2,758	58
Kortschnoj, V L	1	2	50%	2672	0.558	0.292	0.150	22.860	28.550	1,106	23

years: 1962-1970 *PANEL D*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Fischer, Robert James	12	16	75%	.	0.641	0.286	0.073	18.405	28.622	10,706	252
Kortschnoj, V L	7	12	58%	.	0.469	0.459	0.072	19.964	26.378	7,728	197
Keres, Paul	4	10	40%	.	0.420	0.547	0.032	19.080	25.084	4,830	139
Spassky, Boris Vasilievich	4	10	40%	.	0.410	0.570	0.020	18.240	24.341	4,365	138
Botvinnik, Mikhail	3	5	60%	.	0.529	0.414	0.056	18.769	26.065	2,251	63
Geller, Efim P	2	12	16%	.	0.425	0.506	0.069	18.519	25.024	8,432	220
Tal, Mihail	2	10	20%	.	0.466	0.402	0.133	21.894	26.928	4,615	123
Petrosian, Tigran V	2	14	14%	.	0.334	0.621	0.045	20.311	24.675	7,622	227
Reshevsky, Samuel H	1	13	7%	.	0.258	0.505	0.237	24.871	25.505	5,253	140
Bronstein, David Ionovich	0	7	0%	.	0.283	0.628	0.089	21.604	25.199	3,043	94

Table 1 (cont): World's Top 10 chess players and their tournament performances

years: 2014-2019 — *PANEL E*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Hou, Yifan	4	4	100%	2644	0.614	0.364	0.023	16.222	26.820	4,028	88
Ju, Wenjun	3	6	50%	2563	0.400	0.523	0.077	16.565	26.079	5,826	130
Koneru, Humpy	2	6	33%	2584	0.379	0.424	0.197	20.033	27.516	5,832	132
Dzagnidze, Nana	0	6	0%	2540	0.359	0.347	0.295	24.728	28.335	6,080	128
Goryachkina, Aleksandra	0	1	0%	2564	0.364	0.636	0.000	14.322	27.547	1,116	22
Kosteniuk, Alexandra	0	7	0%	2532	0.297	0.508	0.195	22.841	29.092	6,878	150
Lagno, Kateryna	0	2	0%	2544	0.227	0.682	0.091	16.283	26.304	1,666	44
Muzychuk, Anna	0	5	0%	2554	0.218	0.582	0.200	18.416	26.901	4,814	110
Muzychuk, Mariya	0	3	0%	2544	0.242	0.576	0.182	18.302	27.178	3,056	66
Zhao, Xue	0	6	0%	2519	0.288	0.424	0.288	24.058	27.521	5,990	132

Notes: Panels A-E show the tournament performance for the World's Top 10 chess players for the corresponding time period. Elo rating system was first adopted by the International Chess Federation (FIDE) in 1970, hence this information is absent in Panel D for Fischer's sample. Variables and their definitions are presented in table Table A.1.

3.3 ChessBase Mega Database

Our data comes from the 2020 ChessBase Mega Database containing over 8 million chess games dating back to the 1400s. Every chess game is contained in a PGN file which includes information about player names, player sides (White or Black), Elo ratings, date and location of the game, tournament name, round, and the moves played. An example of a PGN file and a tournament table is provided in the appendix. See Table A.2 and Figure A.1.

Table A.1 in the appendix provides a summary of variables used and their definitions. Table 2 presents the summary statistics for each era with tournaments grouped according to the superstar presence. In total, our study analyzes over 2 million moves from approximately 35,000 games played in over 300 tournaments between 1962 and 2019.[29]

3.4 Measuring Performance

3.4.1 Average Centipawn Loss

Our first metric comes from computer evaluations where we identify mistakes committed by each player in a given game.[30] A chess game g consists of moves $m \in \{1, \ldots, M\}$ where player i makes an individual move m_{ig}. A chess engine can evaluate a given position by calculating layers with depth n at each decision node and make suggestions about the best moves to play. Given a best move is played, the engine provides the relative (dis)advantage in a given position $C_{igm}^{computer}$. This evaluation is then compared to the actual evaluation score C_{igm}^{player} once a player makes his or her move. The difference in scores reached via the engine's top suggested move(s) and the actual move a player makes can be captured by

$$error_{igm} = \left| C_{igm}^{computer} - C_{igm}^{player} \right|. \tag{2}$$

If the player makes a top suggested move, the player has committed zero error, i.e., $C_{igm}^{computer} = C_{igm}^{player}$. We can think of chess as a game of attrition where the player who makes less mistakes eventually wins the game. While staying constant if top moves are played, the evaluation shows an advantage for the opponent if a player commits a mistake by playing a bad move.

[29] A list of the tournaments is provided in the appendix.

[30] Guid and Bratko (2006) and Regan et al. (2011) are two early examples of implementations of computer evaluations in chess.

We then take the average of all the mistakes committed by player i in game g via

$$\overline{error}_{ig} = \frac{\sum_{m=1}^{M} \left| C_{igm}^{computer} - C_{igm}^{player} \right|}{M}, \tag{3}$$

which is a widely accepted metric named Average Centipawn Loss (ACPL). ACPL is the average of all the penalties a player is assigned by the chess engine for the mistakes they committed in a game. If the player plays the best moves in a game, his ACPL score will be small where a smaller number implies the player performed better. On the other hand, if the player makes moves that are considered bad by the engine, the player's ACPL score would be higher.

We used Stockfish 11 in our analysis with depth $n = 19$ moves.[31] For each move, the engine was given half a second to analyze the position and assess $|C_{igm}^{computer} - C_{igm}^{player}|$. Figure A.8 shows an example of how a game was analyzed. For instance, at move 30, the computer evaluation is +3.2, which means that the white player has the advantage by a score of 3.2: roughly the equivalent of being one piece (knight or bishop) up compared to his opponent. If the white player comes up with the best moves throughout the rest of the game, the evaluation can also stay 3.2 (if the black player also makes perfect moves) or only go up leading to a possible win toward the end of the game. In the actual game, the player with the white pieces lost his advantage by making bad moves and eventually lost the game. The engine analyzes all 162 moves played in the game and evaluates the quality of each move. Dividing the sum of mistakes committed by player i to the total number of moves played by player i gives the player-specific ACPL score.

3.4.2 Board Complexity

Our second measure that reinforces our ACPL metric is "board complexity" which we obtain via an Artificial Neural Network (ANN) approach. The recent developments with AlphaGo and AlphaZero demonstrated the strength of using heuristic-based algorithms that perform at least as good as the traditional approaches, if not better.[32] Instead of learning from self-play, our neural-network algorithm "learns" from human players.[33] To train the network, we use an independent sample published as part of a Kaggle contest consisting of 25,000 games and more than 2 million moves, with Stockfish

[31] While it is possible to consider higher depths, $n = 19$ depth is more than sufficient for Stockfish to make accurate searches. See Guid and Bratko (2017) who consider sensitivity in depth levels 2-12, or Backus et al. (2016) who use depth level 15.

[32] https://en.chessbase.com/post/leela-chess-zero-alphazero-for-the-pc

[33] Sabatelli et al. (2018) and McIlroy-Young et al. (2020) are two recent implementations of such architecture.

evaluation included for each move.[34] The average player in this sample has an Elo rating of 2280, which corresponds to the "National Master" level according to the United States Chess Federation (USCF).[35]

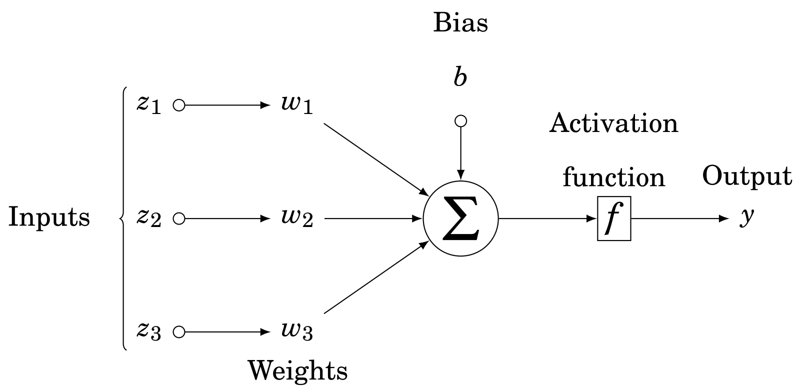

Figure 3: Example of a simple perceptron, with 3 input units (each with its unique weight) and 1 output unit.

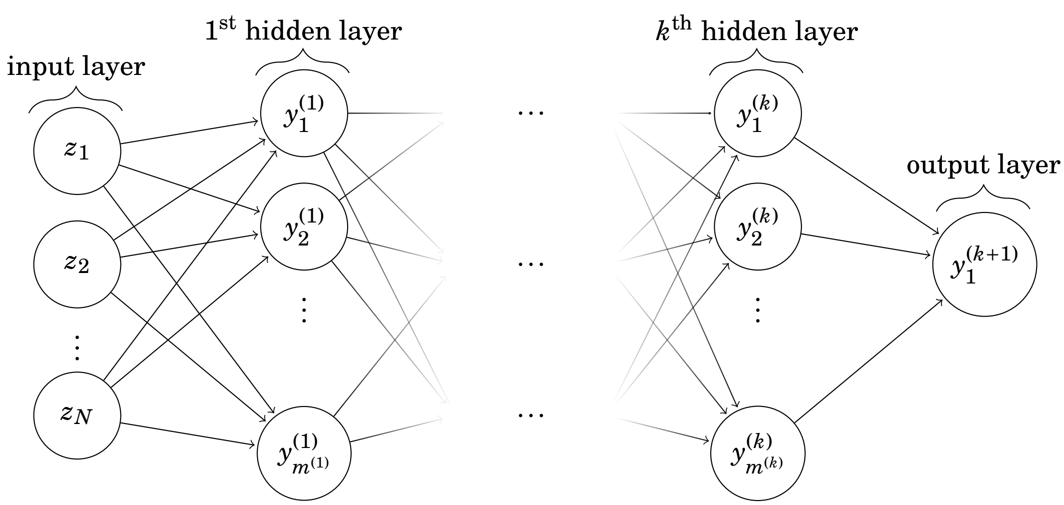

Figure 4: Network graph of a multilayer neural network with $(k+1)$ layers, N input units, and 1 output unit. Each neuron collects unique weights from each previous unit. The k^{th} hidden layer contains $m^{(k)}$ neurons.

The goal of the network is to predict the probability of a player making a mistake with its magnitude. This task would be trivial to solve for positions that were previously played. However, each chess game reaches a unique position after the opening stage which requires accurate extrapolation of human play in order to predict the errors.[36] We represent a chess position through the use of its

[34] https://www.kaggle.com/c/finding-elo/data

[35] http://www.uschess.org/index.php/Learn-About-Chess/FAQ-Starting-Out.html

[36] This approach is vastly different than traditional analysis with an engine such as Stockfish. Engines are very strong and can find the best moves. However, they cannot give any information about how a human would play in a given situation because they are designed to find the best moves without any human characteristics. Our neural-network algorithm is specifically designed to learn how and when humans make mistakes in given positions from analyzing mistakes committed by humans from a sample of 2 million moves.

12 binary features, corresponding to the 12 unique pieces on the board (6 for White, 6 for Black). A chess board has $8 \times 8 = 64$ squares. We split the board into 12 separate 8×8 boards (one for each piece) where a square gets "1" if the piece is present on that particular square and gets "0" otherwise. In total, we represent a given position using $12 \times 8 \times 8 = 768$ inputs. We add one additional feature to represent the players' turn (white to move, or black to move) and thus have $768 + 1 = 769$ inputs in total per position. Figure 4 illustrates. [37]

The neural network "learns" from 25,000 games by observing each of the approximately two million positions and estimates the optimal weights by minimizing the error rate that results from each possible set of weights with the Gradient Descent algorithm. The set of 1,356,612 optimal weights uniquely characterizes our network. We use two networks to make a prediction on two statistics for a given position: (i) probability that a player commits an error and (ii) the amount of error measured in centipawns. For a full game, the two statistics multiplied (and averaged out across moves) gives us an estimate for the ACPL that each player is expected to get as the result of the complexity of the game

$$E(\overline{error_{ig}}) = \frac{\sum_{m=1}^{M} P\left(\left|C_{igm}^{computer} - C_{igm}^{network}\right| > 0\right)\left|C_{igm}^{computer} - C_{igm}^{network}\right|}{M}, \tag{4}$$

where $\left|C_{igm}^{computer} - C_{igm}^{network}\right|$ is the expected centipawn loss in a given position predicted by the neural network. We test our network's performance on our main "superstars" sample.[38] The mean ACPL for the whole sample with 35,000 games is 25.87, and our board complexity measure, which is the expected ACPL that we obtained through our network, is 26.56.[39] Figure A.11 shows a scatterplot of ACPL and the expected ACPL. The slope coefficient is 1.14, which implies that a point increase in our complexity measure results in a 1.14 point increase in the actual ACPL score.[40] Figure A.12 shows the distributions of ACPL and the expected ACPL.

The board complexity measure addresses the main drawback of using only ACPL scores. The

[37] We use a network architecture with three layers. Figure 3 illustrates. The layers have 1048, 500, and 50 neurons, each with its unique weight. In order to prevent overfitting, a 20% dropout regularization on each layer is used. Each hidden layer is connected with the Rectified Linear Unit (ReLU) activation function. The Adam optimizer was used with a learning rate of 0.001.

[38] Figures A.9–A.10 show our network's prediction on a game played by Magnus Carlsen.

[39] The reason why our network –which was trained with games played at on average 2280 Elo level– makes a close estimate for the ACPL in the main sample is that the estimates come from not a single player with Elo rating 2280, but rather from a "committee" of players with Elo rating 2280 on average. Hence the network is slightly "stronger" compared to an actual 2280 player.

[40] The highest ACPL prediction of the network is 50.2 while about 8% of the sample has an actual ACPL > 50.2. These extreme ACPL cases are under-predicted by the network due to the network's behavior as a "committee" rather than a single player, where the idiosyncratic shocks are averaged out.

ACPL score of a player is a function of his or her opponent's strength and their strategic choices. For instance, if both players find it optimal to not take any risks, they can have a simple game where players make little to no mistakes, resulting in low ACPL scores. Yet, this would not imply that players showed a great performance compared to their other −potentially more complex− games. Being able to control for complexity of a game enables us to compare mistakes committed in similarly-complex games.

3.4.2 Game outcomes

The third measure we use is game-level outcomes. Every chess game ends in a win, a loss, or a draw. The player who wins a tournament is the one who accumulates more wins and fewer losses, as the winner of a game receives a full point toward his or her tournament score.[41] In other words, a player who has more wins in a tournament shows a higher performance. In terms of losses, the opposite is true. If a player has many losses in a tournament, their chances to win the tournament are slim. Of course, a draw is considered better than a loss and worse than a win.

4. Empirical Design

Our baseline model compares a player's performance in a tournament where a superstar is present with their performance in a tournament without a superstar. This can be captured by the following equation:

$$\begin{aligned} Performance_{i,j} = \beta_0 &+ \beta_1 Superstar_j \times HighELO_{i,j} + \beta_2 Superstar_j \times MidELO_{i,j} \\ &+ \beta_3 Superstar_j \times LowELO_{i,j} + \beta_4 HighELO_{i,j} + \beta_5 MidELO_{i,j} \\ &+ \mathbf{\Theta} \boldsymbol{X_{i,j}} + \eta_i + \epsilon_{i,j}, \quad (5) \end{aligned}$$

where $Performance_{i,j}$ is the performance of player i in tournament j, measured by methods discussed in section 3.4. $Superstar_j$ is an indicator for a superstar being present in tournament j. $\epsilon_{i,j}$ is an idiosyncratic shock. Having negative signs for coefficients β_1, β_2, and β_3 means that the superstar presence creates an adverse effect: players are discouraged to demonstrate their best efforts resulting in worse performance outcomes. $HighELO_{i,j}$ equals one if the player has an Elo rating within the top quartile in the Elo rating distribution of the tournament participants. $MidELO_{i,j}$

[41] A draw brings half a point, while a loss brings no points in a tournament.

captures the second and third quartiles, and $LowELO_{i,j}$ captures the bottom quartile. $\mathbf{\Theta X_{i,j}}$ contains the game and tournament level controls. In addition to tournament level specifications, chess allows for a game level analysis which can be specified as the following:

$$Performance_{i,j,k} = \alpha_0 + \alpha_1 AgainstSuperstar_{i,j,k} + \mathbf{\Phi X_{i,j}} + \eta_i + \epsilon_{i,j,k}, \tag{6}$$

where $AgainstSuperstar_{i,j,k}$ equals one if player i in tournament j plays against a superstar in round k. In this specification, α_1 captures the effect of head-to-head competition against a superstar.

Note that chess superstars as a rule play in the strongest tournaments, which guarantees more money and higher prestige. However, it is not possible to play in all top-level tournaments.[42] Typically, if a player competes in a world championship match in a given year, (s)he tends to play in fewer tournaments in that particular year to be able to prepare for the world championship match.[43] In years without a world championship match, the world champion typically picks a certain number of tournaments to participate in. (S)he may play in fewer tournaments if (s)he believes that the schedule does not allow for adequate preparation for each tournament. We control for the average Elo rating in a tournament to account for any selection issues.[44]

5. Results

Table 3 shows the performance of non-superstar players playing against a superstar for each sample. There is a distinct pattern that is true for all superstars: playing against them is associated with a higher ACPL score, more blunders, more mistakes, lower chances to win, and higher chances to lose. What is more, games played against superstars are more complex. This higher complexity could be due to the superstar's willingness to reach more complex positions in order to make the ability-gap more salient. It could also be linked to a non-superstar player taking more risk.[45] Taken

[42] In our sample with elite tournaments, a tournament with a superstar has, on average, an average Elo score that is 50 points higher compared to the tournaments without a superstar. This shows that chess superstars indeed play in the strongest tournaments.

[43] We indeed document a negative correlation between the number of tournaments a superstar plays and world championship years. These results are available upon request.

[44] Linnemer and Visser (2016) document self-selection in chess tournaments with stronger players being more likely to play in tournaments with higher prizes. A central difference between their sample and ours is the level of tournaments, with their data coming from the World Open tournament, which is an open tournament with non-master participants with Elo ratings between 1400-2200. Meanwhile, our sample consists of players from a much more restricted sample with only the most elite Grandmasters having Elo ratings often above 2700. Moreover, these high-level tournaments are invitation based; i.e., tournament organizers offer invitations to a select group of strong players, with these restrictions working against any possible selection issues.

[45] It is not a trivial task to identify which player initiates complexity. Typically, complex games are reached with mutual agreement by players, avoiding exchanges and keeping the tension on the board.

as a whole, players commit more blunders and mistakes, holding board complexity constant. For instance, a player who plays against Fischer shows an ACPL that is 4.3 points higher compared to his games against other players with a similar complexity level. His likelihood is 10 percentage points less for a win, 18 percentage points less for a draw, and 29 percentage points higher for a loss compared to his typical games. This implies that in terms of direct competition, these superstars have a strong dominance over their peers. Hou Yifan shows the strongest domination, with Fischer closely following behind. The magnitudes for ACPL, win, and loss probabilities are stronger for these players compared to the rest of the samples.[46]

Tables A.4–A.9 show the effect of a superstar's presence on the performance of other competitors. An adverse effect on the top players exists for the most dominant superstar of our sample, Hou Yifan.[47] Her presence is associated with an ACPL score that is 4.3 points higher, 11 percentage points less chances of winning, and 15 percentage points higher chances of losing for the top players in a tournament. For Fischer, the coefficients for ACPL, blunders, and mistakes are positive, and stronger for the top players of his era. Fischer's opponents indeed had more draws and less complex-games, which agrees with the findings in Moul and Nye (2009) on Soviet collusion.

Another situation with intense competition is when two superstars, Kasparov and Karpov, both participate in a tournament. This means that for a given player, he or she will have to face both Kasparov and Karpov and perform better than both of them in order to win the tournament. This tough competition appears to lead to more decisive games and less draws, with players committing fewer blunders and mistakes. The top quartile of players, who try to compete with Kasparov and Karpov, experience high pressure and as a result, experience more losses. Despite these players also win more games, it does not offset their losses.[48]

Players perform better if they face only Kasparov or Karpov in the tournament compared to facing both superstars in the tournament. With one superstar, either Kasparov or Karpov, in the tournament, players have higher chances of winning the tournament and as a result, play more accurately and manage to get more wins, with substantial gains in the ACPL score and less mistakes committed. This improvement is the strongest for the top quartile of players.

Similar to facing Kasparov or Karpov alone, Carlsen's presence creates a slight positive effect on

[46] Taken as a whole, these findings verify that the superstars considered in our study indeed show greater performance compared to their peers.

[47] A potential explanation for why the most dominant superstar in our sample is a female chess player could be related to the Central Limit Theorem. There are much less female than male chess players. A smaller sample has higher variance, making it more likely to produce outliers. See Bilalić et al. (2009) for a discussion of this phenomenon.

[48] In fact, as a spillover, the bottom quartile of players show better performance with Kasparov and Karpov's presence. These players have more wins and fewer losses as a result of a worsened performance by the upper quartile.

his competitors' performance. Players play more accurately and make fewer mistakes under higher challenges with more complex positions. The positive effect appears to apply to all participants.[49]

Table 4 shows the impact of superstar presence for all samples aggregated. Table 5 and Figure 5 show the aggregate superstar effect broken down to each sub-sample for the top quartile players. Moving from Carlsen to Hou Yifan, we observe increases in the committed mistakes, in game complexity, in blunders and mistakes, and in the loss rates. At the same time, the win and draw rates decrease, confirming our theory that an increase in the intensity of a superstar is associated with stronger adverse responses from the top players. When the gap between the superstar and the rest of the group is not too wide, all players perform better. As the gap widens, performance drops are anticipated.

6. Conclusion

The empirical superstar literature finds both positive and negative superstar effect evidence. For example, in golf, the effort level decreases with the superstar presence. However, in the 100-meter running or swimming contests, the effort level appears to increase. In this paper, we show theoretically and empirically that the superstar effect depends on the intensity of the superstar: if the skill gap between the superstar and the rest of the field is small, then the superstar effect is positive; otherwise, if the skill gap is large, the superstar effect is negative.

Using the chess data, we empirically show that the direct superstar (head-to-head) effect is always negative and the indirect superstar effect depends on the skill gap. We find a strong choking effect in head-to-head games with a superstar. Players commit more mistakes than they are expected to when they face a superstar in a head-to-head game. These findings for five chess superstars are consistent with Brown (2011)'s findings for Tiger Woods.

The takeaway for firms seeking to hire a superstar employee is that hiring a superstar employee may create a positive or an adverse effect on the cohort's performance depending on the skill level gap. If the gap is too large, there is a negative effect of hiring a superstar employee. In these settings, a highly skilled team member would hurt competition and create an adverse effect on the rest of the team members. Such adverse effect can occur in many environments. For example, in a classroom, a superstar student may discourage other students from learning in a competitive setting. Maintaining a healthy competition is the key for a productive environment.

[49] Draw rates are higher in tournaments with Carlsen. Many chess fans criticize modern chess for excessive amount of draws, which we document in our analysis.

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Tables & Figures

Table 2: Summary statistics for all samples.

years: 2013-2019					*years: 1995-2001*			
	with Carlsen		*without Carlsen*		*with Kasparov*		*without Kasparov*	
variable	mean	sd	mean	sd	mean	sd	mean	sd
ACPL	16.580	10.755	17.818	11.783	20.554	12.687	21.233	12.947
Difficulty	26.784	5.309	27.040	5.419	27.792	5.576	27.300	5.528
TotalBlunder	.179	.508	.229	.576	.238	.562	.288	.686
TotalMistake	1.432	1.797	1.682	1.943	1.772	1.892	1.876	2.006
win	.173	.378	.204	.403	.230	.421	.229	.421
draw	.654	.476	.592	.491	.545	.498	.541	.498
loss	.173	.378	.204	.403	.224	.417	.229	.420
ELO	2759	47.13	2714	80	2684	59.36	2642	65.52
Moves	43.031	15.682	45.198	17.682	38.551	16.280	38.765	15.656
#of tournaments	=35		=37		=22		=43	
#of games	=1,336		=1,774		=1,727		=3,696	
#of moves	=114,898		=160,362		=133,184		=286,787	
years: 1987-1994					*years: 1976-1983*			
	with Kasparov & Karpov		*without Kasparov & Karpov*		*with Karpov*		*without Karpov*	
ACPL	20.762	12.068	21.592	13.235	21.777	13.501	23.099	14.091
Difficulty	27.139	5.555	26.850	5.811	25.461	5.873	26.007	5.787
TotalBlunder	.264	.603	.317	.684	.271	.639	.327	.727
TotalMistake	1.821	1.902	1.922	2.028	1.850	2.040	2.061	2.089
win	.221	.415	.234	.423	.223	.416	.251	.433
draw	.561	.496	.533	.499	.553	.497	.499	.500
loss	.218	.413	.234	.423	.224	.417	.250	.433
ELO	2629	60.68	2590	55.97	2558	68.05	2531	76.24
Moves	38.917	16.898	39.635	16.810	36.699	17.542	37.964	17.118
#of tournaments	=11		=37		=32		=39	
#of games	=635		=1,989		=1,967		=3,641	
#of moves	=50,212		=157,668		=144,633		=278,876	

Notes: Superstar player observations are exluded in each sample. Data comes from Chessbase Mega Database 2020.

Table 2: Summary statistics for all samples. (cont.)

	years: 1962-1970				*years: 2014-2019*			
	with Fischer		*without Fischer*		*with Hou Yifan*		*without Hou Yifan*	
ACPL	24.017	15.634	25.284	15.670	22.466	14.500	21.128	12.301
Difficulty	25.835	5.599	26.077	5.747	27.193	5.225	27.162	5.045
TotalBlunder	.342	.768	.349	.746	.405	.736	.380	.755
TotalMistake	2.173	2.128	2.311	2.254	2.341	2.344	2.267	2.209
win	.254	.435	.250	.433	.270	.444	.241	.428
draw	.492	.500	.500	.500	.459	.499	.519	.500
loss	.254	.435	.250	.433	.270	.444	.241	.428
ELO	.	.	.	.	2493	72.50	2499	43.43
Moves	38.126	16.458	36.112	15.628	45.823	19.041	46.179	17.511
#of tournaments	=15		=82		=4		=6	
#of games	=1,660		=7,832		=440		=748	
#of moves	=126,578		=565,611		=40,324		=69,084	

Notes: Superstar player observations are exluded in each sample. Data comes from Chessbase Mega Database 2020.
$*$: Elo rating system was first adopted by FIDE beginning 1970.

Table 3: Performance against a superstar.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty	*# of games*	*# of moves*
Against Carlsen	1.868***	0.080***	0.174**	-0.069***	-0.100***	0.169***	0.172	3,316	294,876
2013-2019	(0.449)	(0.026)	(0.085)	(0.015)	(0.023)	(0.025)	(0.252)		
Against Kasparov	2.300***	0.093**	0.226**	-0.104***	-0.049	0.153***	1.485***	5,770	446,322
1995-2001	(0.634)	(0.046)	(0.095)	(0.012)	(0.035)	(0.037)	(0.346)		
Against Kasparov/Karpov	2.757***	0.174***	0.184*	-0.102***	-0.078***	0.180***	-0.427	2,768	219,607
1987-1994	(0.656)	(0.033)	(0.093)	(0.016)	(0.027)	(0.030)	(0.326)		
Against Karpov	3.171***	0.167***	0.150*	-0.106***	-0.089***	0.195***	0.522**	5,326	396,903
1976-1983	(0.579)	(0.036)	(0.083)	(0.012)	(0.024)	(0.024)	(0.253)		
Against Fischer	4.379***	0.150***	0.222	-0.106***	-0.186***	0.292***	2.255***	9,626	703,525
1962-1970	(0.949)	(0.040)	(0.149)	(0.022)	(0.032)	(0.040)	(0.361)		
Against Hou Yifan	4.415**	0.203***	0.502	-0.111***	-0.203***	0.314***	0.839**	1,232	113,436
2014-2019	(1.530)	(0.047)	(0.437)	(0.031)	(0.034)	(0.029)	(0.331)		
Against Superstar	2.523***	0.101***	0.105*	-0.0917***	-0.0750***	0.167***	0.639***	27,854	2,174,669
1962-2019	(0.312)	(0.016)	(0.055)	(0.008)	(0.014)	(0.015)	(0.155)		

Notes: All regressions include player and year fixed effects, round fixed effects, event site fixed effects, board complexity measured by our neural-network algorithm (except in column 7), opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Performance in tournaments with and without a Superstar (overall effect).

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
Top 25% players	-0.326	-0.037*	0.069	-0.006	0.011	-0.005	0.368**
	(0.352)	(0.019)	(0.054)	(0.011)	(0.015)	(0.015)	(0.174)
Mid 50% players	-0.585**	-0.022	-0.062*	0.009	-0.001	-0.008	0.169
	(0.243)	(0.015)	(0.035)	(0.008)	(0.011)	(0.009)	(0.138)
Bottom 25% players	-0.029	-0.042*	0.046	0.003	0.008	-0.011	0.063
	(0.381)	(0.022)	(0.047)	(0.011)	(0.013)	(0.013)	(0.165)
Number of moves	1,216,450	1,216,450	1,216,450	1,216,450	1,216,450	1,216,450	1,216,450
Number of games	20,295	20,295	20,295	20,295	20,295	20,295	20,295

Notes: Superstars' games are excluded. Top 25% is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an Elo in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament, player's Elo rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Performance in tournaments with and without a superstar for the top players.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty	*# of games*	*# of moves*
Carlsen present	-0.080	0.040	-0.128	0.010	0.022	-0.032	0.756**	3,110	275,260
2013-2019	(0.698)	(0.032)	(0.097)	(0.031)	(0.038)	(0.032)	(0.362)		
Kasparov present	-1.427**	-0.086**	-0.142	0.025	0.028	-0.053**	0.694	5,427	420,348
1995-2001	(0.696)	(0.035)	(0.102)	(0.025)	(0.033)	(0.025)	(0.466)		
Kasparov&Karpov present	-0.848	-0.062	-0.124	0.031	-0.082**	0.051	0.377	2,624	207,880
1987-1994	(1.030)	(0.059)	(0.168)	(0.033)	(0.036)	(0.039)	(0.436)		
Karpov present	-1.843	-0.205**	-0.148	-0.050	0.137*	-0.088	2.316**	5,120	381,460
1976-1983	(2.357)	(0.098)	(0.399)	(0.055)	(0.071)	(0.100)	(0.895)		
Fischer present[+]	1.226*	0.054*	0.245**	-0.075**	0.056*	0.018	-0.746***	9,491	692,072
1962-1970	(0.660)	(0.032)	(0.100)	(0.032)	(0.030)	(0.026)	(0.274)		
Hou Yifan present	4.557**	-0.041	0.970***	-0.118**	-0.055	0.172**	0.734	1,188	109,408
2014-2019	(1.999)	(0.112)	(0.341)	(0.058)	(0.057)	(0.071)	(0.752)		
Aggregate effect[++]	-0.326	-0.037*	0.069	-0.006	0.011	-0.005	0.368**	26,960	2,086,428
	(0.352)	(0.019)	(0.054)	(0.011)	(0.015)	(0.015)	(0.174)		

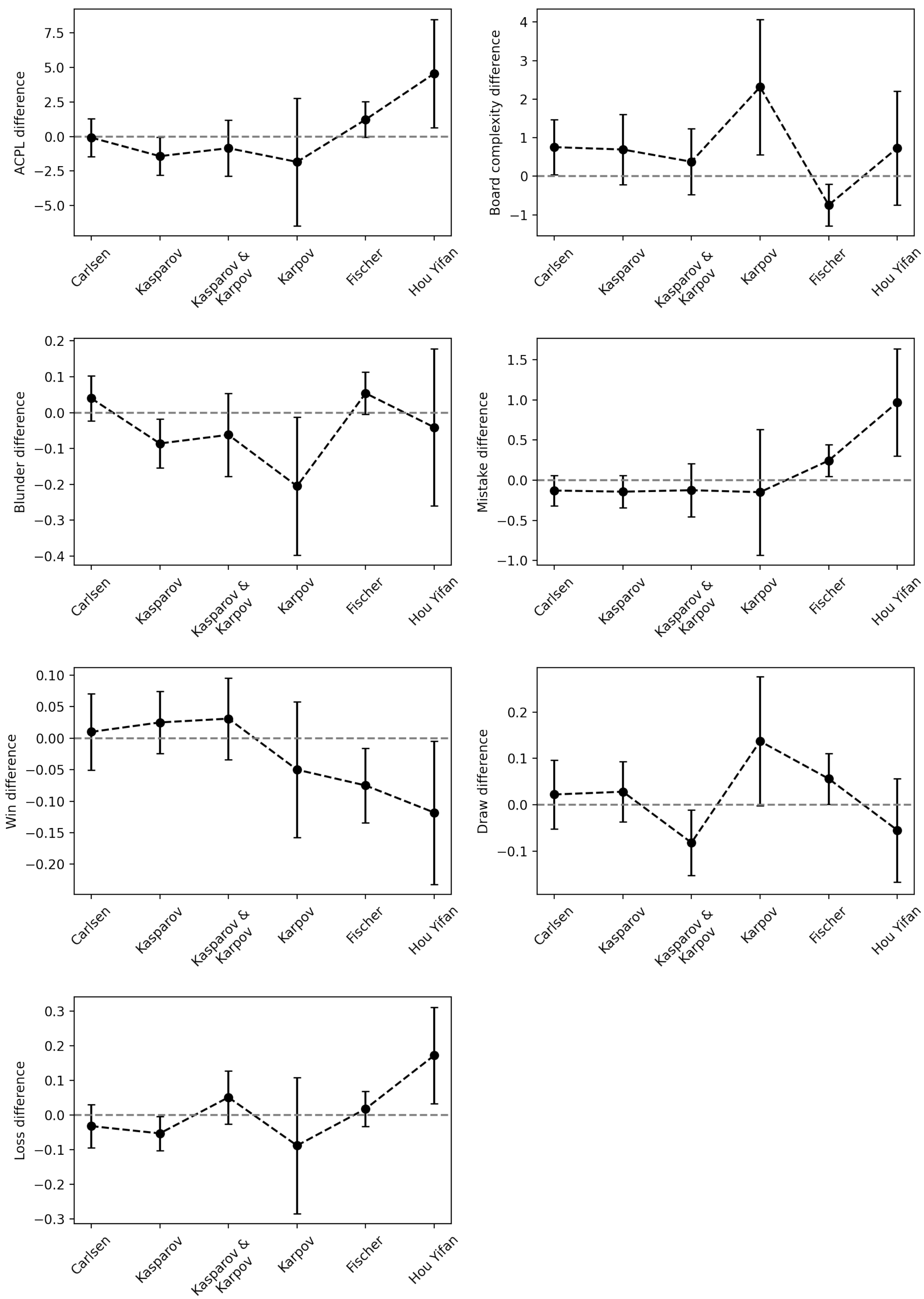
Notes: Superstars' games are excluded. A top player is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament (except for pre-1970 games, as Elo rating was adopted in 1970 by FIDE), player's Elo rating (except pre-1970 games), board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

$^{+}$: Since no Elo rating information was available in Fischer's era, we define the top players as the top chess players in the world from 1962-1970 other than Fischer. These players are Tigran Petrosian, Viktor Korchnoi, Boris Spassky, Vasily Smyslov, Mikhail Tal, Mikhail Botvinnik, Paul Keres, Efim Geller, David Bronstein, and Samuel Reshevsky. Kasparov (2003) provides a detailed overview on each of these players.

$^{++}$: The sample is restricted to tournaments with Elo rating information available.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Figure 5: Superstar presence coefficients for top players over different superstar intensity levels.



Note: The figure presents the coefficients in Table 5 with 95% confidence intervals over different superstar intensity levels.

Appendix (For Online Publication)

Table A.1: Variables list

Variable Name	*Variable Meaning*
Superstar Present	=1 if a superstar is present in a tournament.
Against Superstar	=1 if a game is played against a superstar.
ELO	Elo rating of a player.
ACPL	Average Centipawn Loss of a player in a game.
TotalBlunder	Total number of blunders committed by a player in a game. A move is considered a blunder if the change in centipawn score is more than 300 centipawns.
TotalMistake	Total number of mistakes committed by a player in a game. A move is considered a mistake if the change in centipawn score is between 100-300 centipawns.
Difficulty	The board complexity metric estimated via an Artificial Neural Network algorithm.
win	=1 if a player wins his or her game.
draw	=1 if a games ends in a draw.
loss	=1 if a player loses his or her game.

Table A.1: Variables list (cont.)

Variable Name	*Variable Meaning*
white	=1 if a player's side is white.
moves	Total number of moves played by a player in a game.
Round-robin tournament	An invitation based tournament system with a limited number of participants. Each participant plays against participants once or twice, depending on the tournament length. The participant who accumulates the highest number of points wins the tournament.
Swiss tournament	A tournament system that is typically used in open tournaments with a large pool of participants. Following the results of the first round, winners are paired with other winners. Towards the end of the tournament, strongest players with the highest number of scores get paired. The participant who accumulates the highest number of points wins the tournament.

Table A.2: An example pgn file.

[Event "GRENKE Chess Classic"]
[Site "Karlsruhe GER"]
[Date "2019.04.20"]
[EventDate "2019.04.20"]
[Round "1"]
[Result "0-1"]
[White "Vincent Keymer"]
[Black "Magnus Carlsen"]
[ECO "A56"]
[WhiteElo "2516"]
[BlackElo "2845"]
[PlyCount "162"]

1 d4 ♘f6 2 c4 c5 3 d5 g6 4 ♘c3 d6 5 e4 ♗g7 6 ♘f3 O-O 7 ♗e2 e5 8 O-O ♘e8 9 ♘e1 f5 10 e×f5 g×f5 11 f4 ♘d7 12 ♘d3 e4 13 ♘f2 ♗×c3 14 b×c3 ♘df6 15 ♗e3 ♘g7 16 ♕e1 ♗d7 17 ♘d1 ♗a4 18 h3 ♗×d1 19 ♕×d1 ♕e8 20 ♔f2 ♕g6 21 ♖g1 ♔h8 22 a4 ♖g8 23 ♕f1 ♘fh5 24 g3 ♖af8 25 ♕g2 ♕f6 26 ♖ac1 ♕d8 27 ♕h2 ♘f6 28 g4 ♘d7 29 g5 ♕a5 30 g6 h6 31 ♖b1 ♖b8 32 ♕g3 ♕d8 33 ♔e1 ♘e8 34 ♔d2 ♘f8 35 ♗f2 ♕e7 36 ♔e3 ♕f6 37 ♔d2 ♘×g6 38 h4 ♘e7 39 ♕h3 ♖×g1 40 ♖×g1 ♕f7 41 h5 ♘f6 42 ♗h4 b6 43 ♖b1 ♕f8 44 ♖g1 ♕f7 45 ♖b1 ♕g7 46 ♖g1 ♕f8 47 ♔c2 ♘fg8 48 ♔d2 ♕f7 49 ♔c2 ♖f8 50 ♔d2 ♕e8 51 ♖a1 ♖f7 52 a5 b×a5 53 ♖×a5 ♘c8 54 ♖a1 ♕f8 55 ♖b1 ♘b6 56 ♖g1 ♖g7 57 ♖×g7 ♔×g7 58 ♕g3+ ♔h8 59 ♕g6 a5 60 ♗f1 a4 61 ♔c2 a3 62 ♔b3 ♘a4 63 ♗h3 ♕g7 64 ♕×g7+ ♔×g7 65 ♗×f5 ♘f6 66 ♔×a3 ♘×c3 67 ♗f2 ♘e2 68 ♔a4 ♘×h5 69 ♔a5 ♘f6 70 ♔b6 ♔f7 71 ♔c7 ♔e7 72 ♗e3 ♘d4 73 ♗g6 h5 74 ♗f2 ♘f3 75 ♗f5 ♘d2 76 ♗h4 e3 77 ♗d3 ♘f3 78 ♗×f6+ ♔×f6 79 ♔×d6 h4 80 ♔c7 ♘d4 81 ♔c8 e2 0-1

Figure A.1: An example of a tournament table.

Grenke Chess Classic 6th 2019

			1	2	3	4	5	6	7	8	9	10		TB	Perf.	+/-
1	Carlsen,Magnus	2845	*	½	½	1	½	1	1	1	1	1	7.5 / 9		2990	+14
2	Caruana,Fabiano	2819	½	*	1	½	½	½	½	½	1	1	6.0 / 9		2833	+3
3	Naiditsch,Arkadij	2695	½	0	*	½	1	½	0	½	1	1	5.0 / 9	19.00	2766	+9
4	Vachier Lagrave,Maxime	2773	0	½	½	*	½	½	½	½	1	1	5.0 / 9	18.25	2758	-1
5	Anand,Viswanathan	2774	½	½	0	½	*	½	½	1	0	1	4.5 / 9	19.75	2719	-6
6	Aronian,Levon	2763	0	½	½	½	½	*	1	½	½	½	4.5 / 9	18.75	2720	-5
7	Svidler,Peter	2735	0	½	1	½	½	0	*	½	1	½	4.5 / 9	17.75	2723	-1
8	Vallejo Pons,Francisco	2693	0	½	½	½	0	½	½	*	½	1	4.0 / 9		2689	-1
9	Meier,Georg	2628	0	0	0	0	1	½	0	½	*	0	2.0 / 9	8.75	2518	-12
10	Keymer,Vincent	2516	0	0	0	0	0	½	½	0	1	*	2.0 / 9	6.50	2529	+1

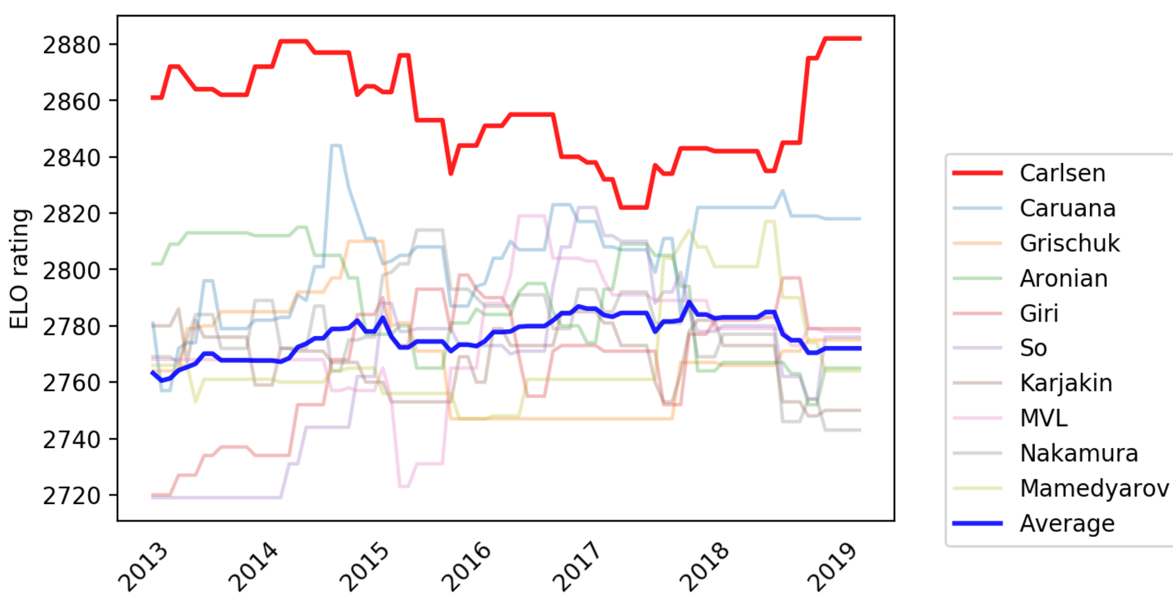
Average Elo: 2724 <=> Cat: 19

gm = 3.24 m = 1.44

(45 Games)

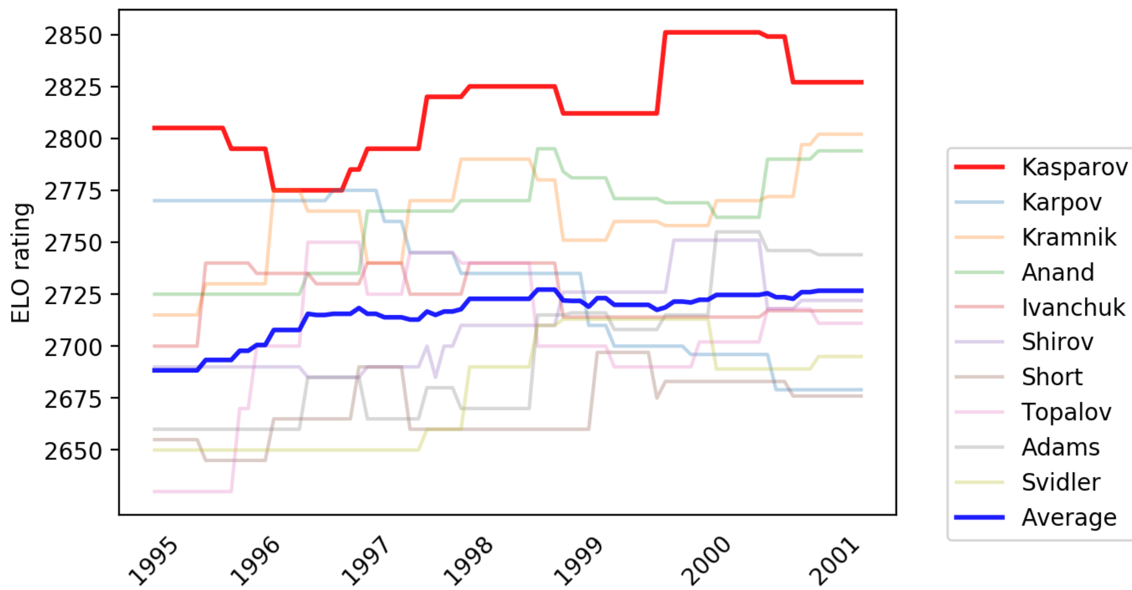
Note: The tournament table is obtained from Chessbase Mega Database 2020.

Figure A.2: Elo ratings of top chess players between 2013-2019.



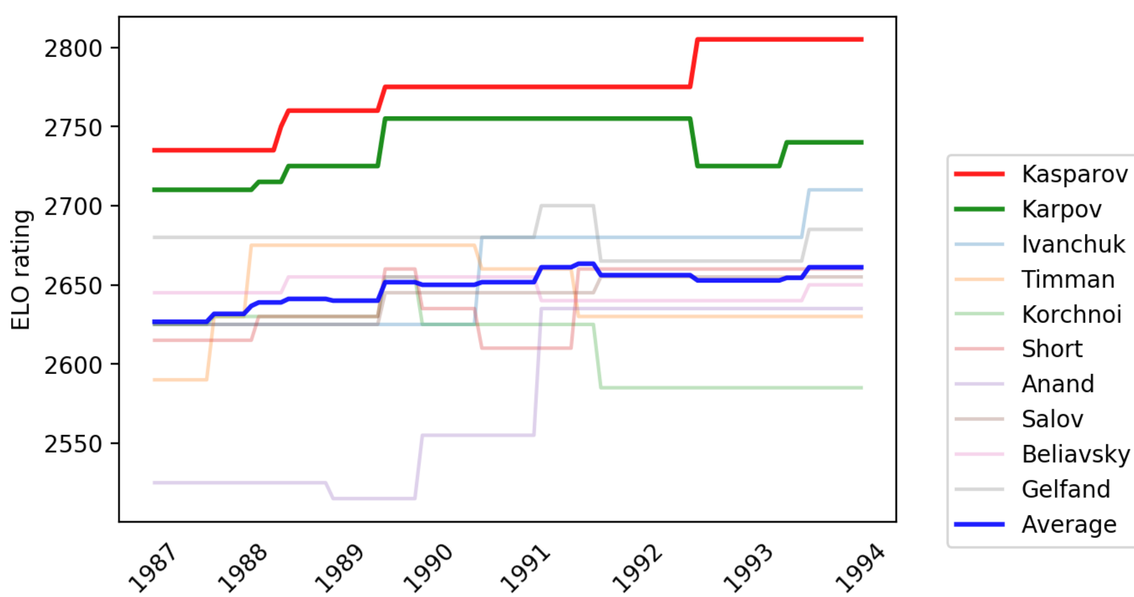
Note: The blue line shows the average Elo rating of top chess players other than Carlsen (World ranking 2–10). Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.3: Elo ratings of top chess players between 1995-2001.



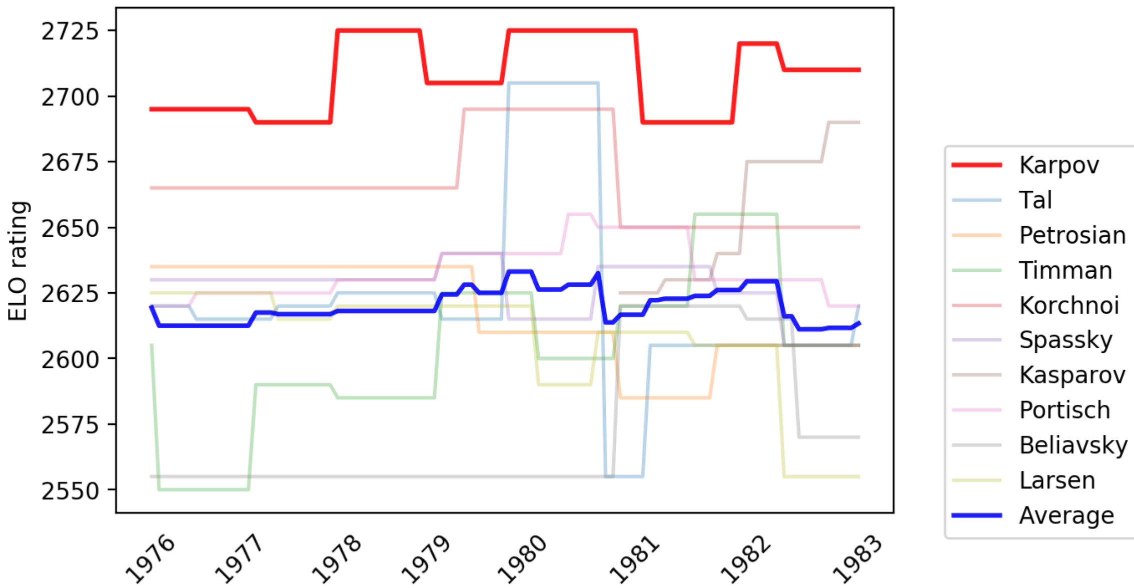
Note: Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.4: Elo ratings of top chess players between 1987-1994.



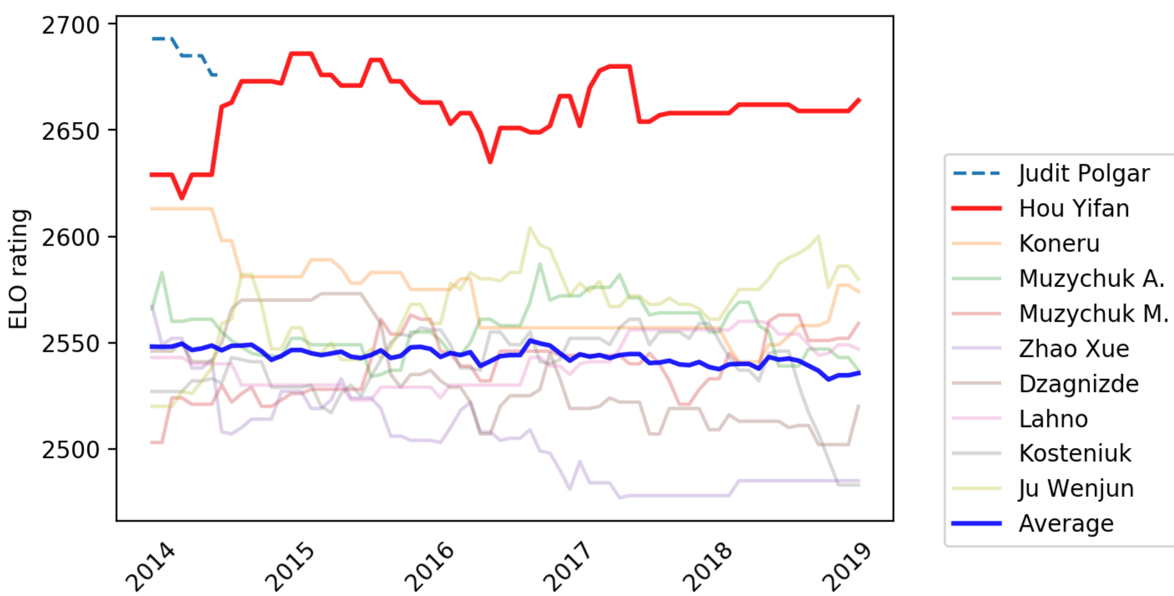
Note: Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.5: Elo ratings of top chess players between 1976-1983.



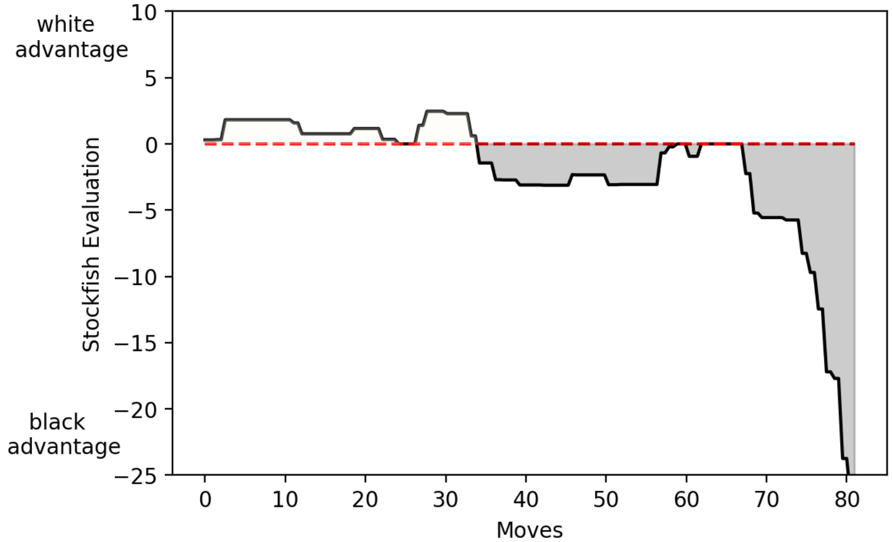
Note: Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.6: Elo ratings of top female chess players between 2014-2019.



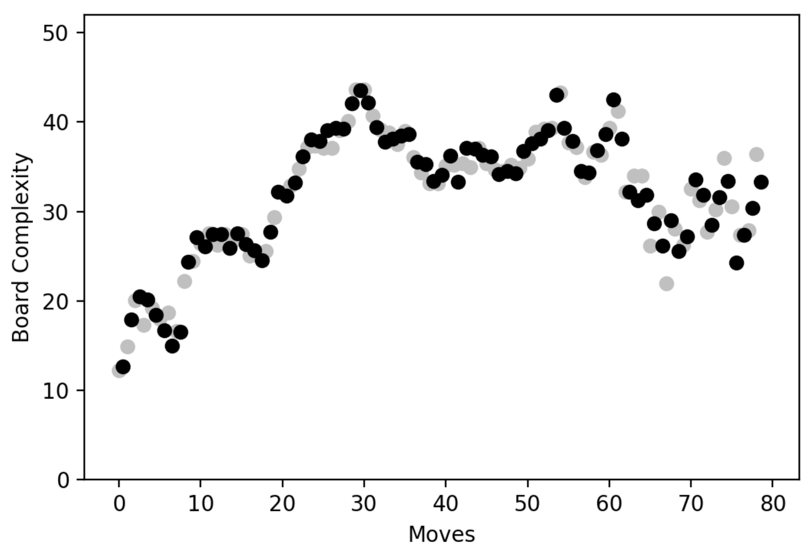
Note: Judit Polgar is considered the strongest female chess player of all time, however she stopped competing in female tournaments in 1990 when she was 14 years old. Hou Yifan stopped competing in female tournaments after 2017. Elo rating data is obtained from FIDE available online at https://ratings.fide.com

Figure A.8: Computer evaluation of a game played by Carlsen in 2019.



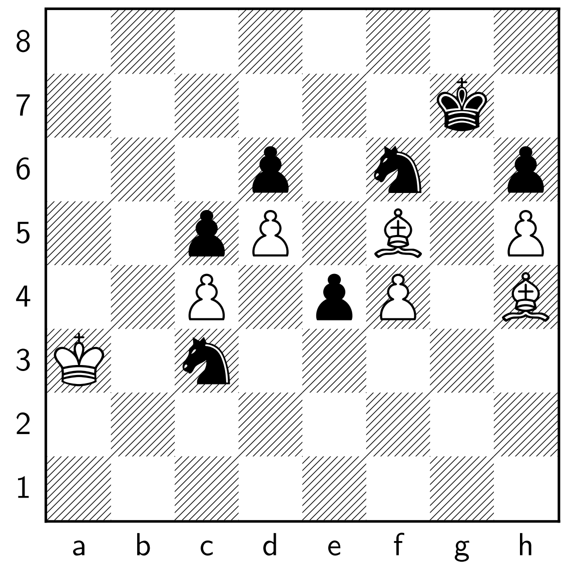
Note: The game was played between Vincent Keymer (White) and Magnus Carlsen (Black) on April 20, 2019 during the first round of Grenke Chess Classic 2019. Keymer's Average Centipawn Loss (ACPL) was 35.22 and Carlsen's 26.17, calculated by using Equation 3. A higher ACPL means the player made more mistakes according to the chess engine. The chess engine used for evaluations is Stockfish 11 with a depth of 19 moves.

Figure A.9: Complexity evaluation of a game played by Carlsen in 2019 using an Artificial Neural Network (ANN) algorithm.



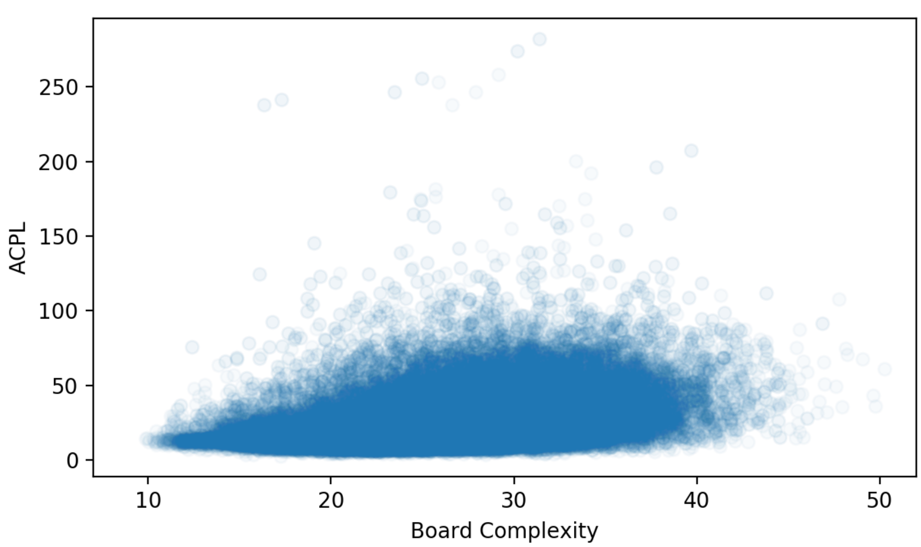
Note: The game was played between Vincent Keymer (White) and Magnus Carlsen (Black) on April 20, 2019 during the first round of Grenke Chess Classic 2019. Keymer's Average Centipawn Loss (ACPL) was 35.22 and Carlsen's 26.17 using our algorithm. Our neural-network board complexity estimate assigns an expected ACPL score of 34.87. This score is substantially higher than the sample average, 26.56. The game is within the top 10% of the sample in terms of complexity.

Figure A.10: A position from Keymer vs. Carlsen (2019).



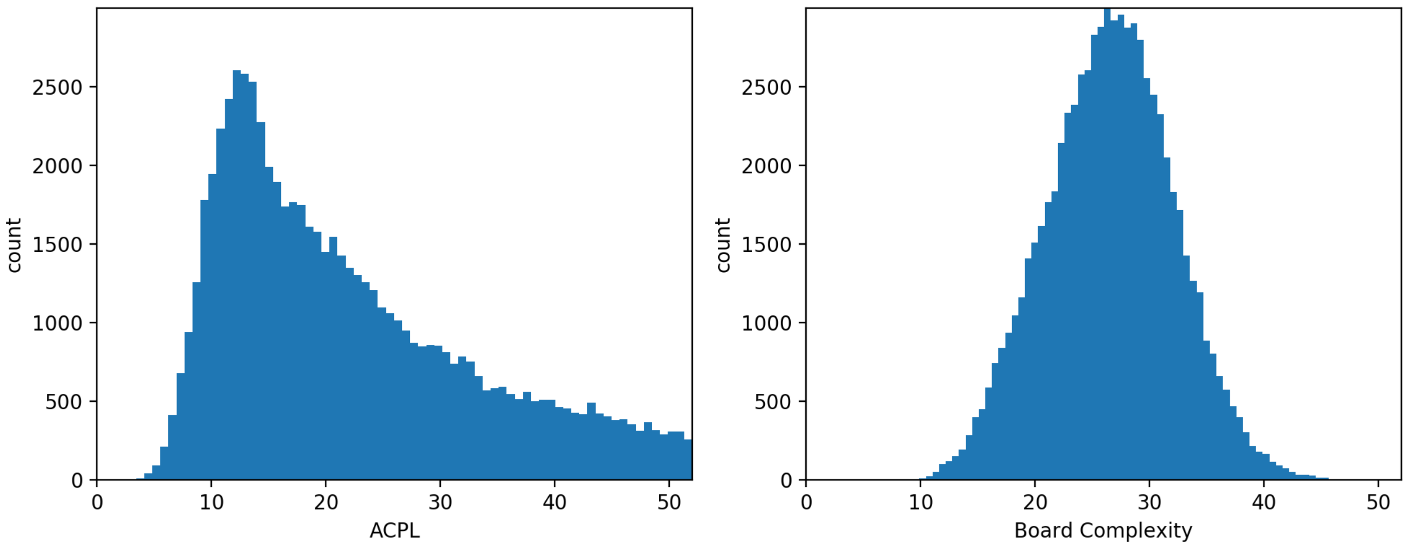
Note: This position is from Vincent Keymer (White) vs. Magnus Carlsen (Black), Grenke Chess Classic 2019 (white to play). Our neural-network algorithm calculates the probability of making an error as 0.52 (about twice as high as the sample average) in an amount of 65 centipawns. In the game, white blundered (by playing Bf2) in an amount of 180 centipawns, according to Stockfish. Before this blunder, the position was a forced draw.

Figure A.11: Scatterplot of board complexity and ACPL scores.



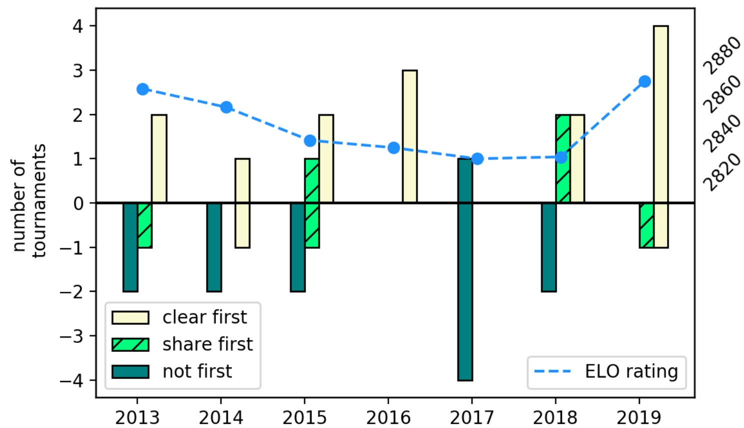
Note: The board complexity measure is obtained via a neural-network algorithm. It is the "expected ACPL score" according to the AI, depending on the complexity of a game. The estimated slope is 1.14 for the overall sample of 32,000 games and 2.1 million moves.

Figure A.12: Distribution of ACPL and board complexity scores.



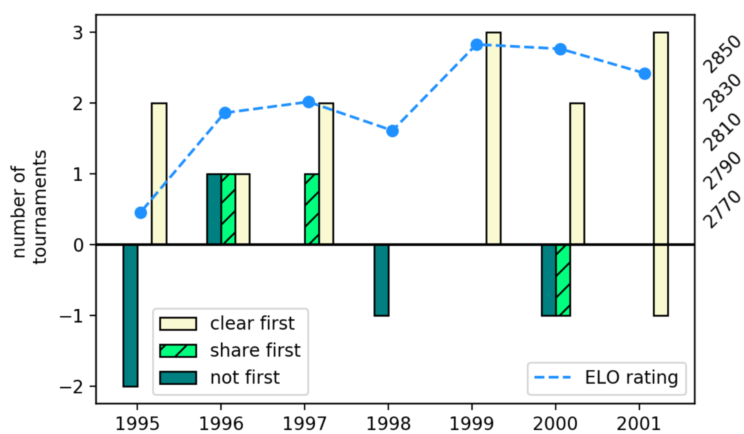
Note: The board complexity measure is obtained via a neural-network algorithm. It is the "expected ACPL score" according to the AI which depends on the complexity of a game. The average ACPL score in the sample is 25.49 and the board complexity score is 26.57 for the overall sample with 32,000 games and 2.1 million moves. The neural-network was trained with an independent sample consisting of 25,000 games and 2 million moves with games played between players with "National Master" ranking on average.

Figure A.13: Carlsen's tournament performance (classical)



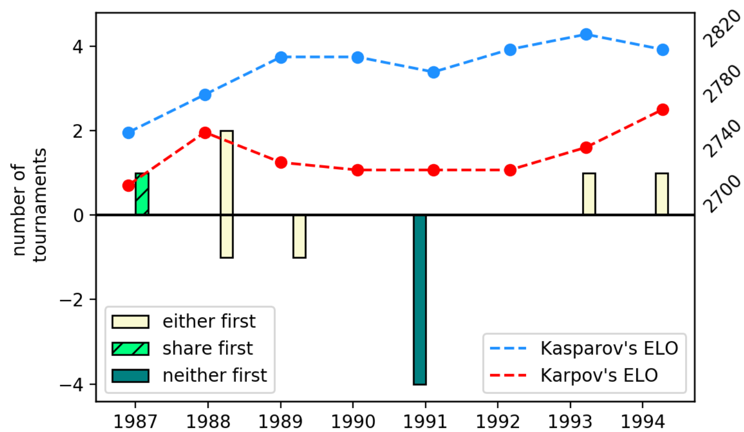
Note: Carlsen's Elo rating data is obtained from FIDE.

Figure A.14: Kasparov's tournament performance (classical)



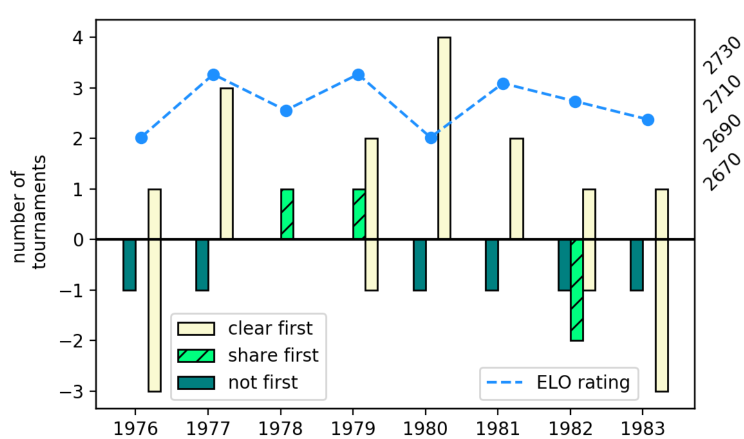
Note: Kasparov's Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.15: Kasparov and Karpov's tournament performance (classical)



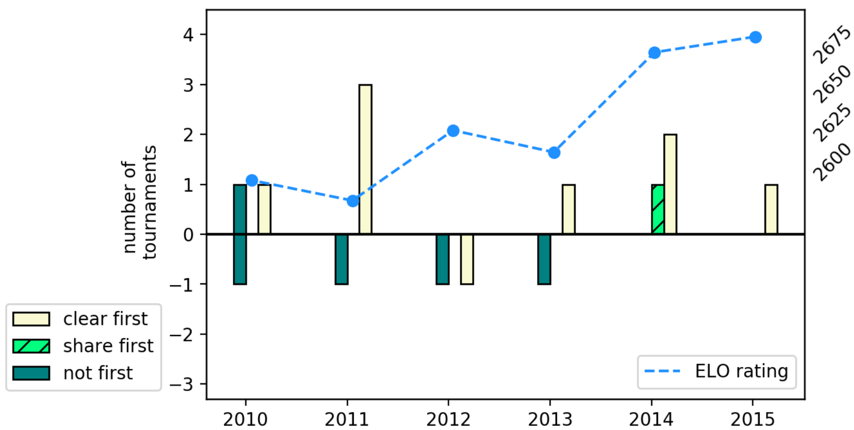
Note: Kasparov's and Karpov's Elo rating data are obtained from Chessbase Mega Database 2020.

Figure A.16: Karpov's tournament performance (classical)



Note: Karpov's Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.17: Hou Yifan's tournament performance (classical)



Note: Hou Yifan's Elo rating data is obtained from Chessbase Mega Database 2020.

Table A.4: Performance in tournaments with and without Hou Yifan.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
Top 25% players	4.344**	-0.012	0.807**	-0.104*	-0.048	0.152**	1.204
	(1.846)	(0.112)	(0.305)	(0.054)	(0.054)	(0.066)	(0.753)
Mid 50% players	1.143	0.015	0.076	-0.068	0.085*	-0.017	1.172**
	(1.039)	(0.083)	(0.207)	(0.042)	(0.047)	(0.056)	(0.447)
Bottom 25% players	1.511	-0.017	0.138	-0.050	-0.032	0.082	0.390
	(1.699)	(0.104)	(0.236)	(0.054)	(0.044)	(0.075)	(0.690)
Number of moves	109,408	109,408	109,408	109,408	109,408	109,408	109,408
Number of games	1,188	1,188	1,188	1,188	1,188	1,188	1,188

Notes: Hou Yifan's games are excluded. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament, player's Elo rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.5: Performance in tournaments with and without Fischer.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
All players	0.465	0.021	0.046	-0.024*	-0.006	0.030**	-0.539***
	(0.368)	(0.017)	(0.061)	(0.014)	(0.012)	(0.015)	(0.153)
Top players	1.226*	0.054*	0.245**	-0.075**	0.056*	0.018	-0.746***
	(0.660)	(0.032)	(0.100)	(0.032)	(0.030)	(0.026)	(0.274)
Number of moves	692,072	692,072	692,072	692,072	692,072	692,072	692,072
Number of games	9,491	9,491	9,491	9,491	9,491	9,491	9,491

Notes: Fischer's games are excluded. Top 10 players are the top chess players in the world from 1962-1970 other than Fischer.[+] All regressions include player and year fixed effects, round fixed effects, event site fixed effects, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

[+]: These players are Tigran Petrosian, Viktor Korchnoi, Boris Spassky, Vasily Smyslov, Mikhail Tal, Mikhail Botvinnik, Paul Keres, Efim Geller, David Bronstein, and Samuel Reshevsky.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.6: Performance in tournaments with and without Kasparov & Karpov.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect *when both present*							
Top 25% players	-0.174	-0.057	-0.034	0.014	-0.072*	0.058	0.692
	(1.048)	(0.052)	(0.185)	(0.031)	(0.038)	(0.041)	(0.452)
Mid 50% players	0.088	-0.006	-0.093	0.021	-0.028	0.007	0.285
	(0.568)	(0.050)	(0.084)	(0.021)	(0.026)	(0.023)	(0.347)
Bottom 25% players	-2.137**	-0.099*	-0.174	0.063**	-0.006	-0.057*	0.316
	(0.935)	(0.049)	(0.146)	(0.025)	(0.036)	(0.031)	(0.418)
Number of moves	207,880	207,880	207,880	207,880	207,880	207,880	207,880
Number of games	2,624	2,624	2,624	2,624	2,624	2,624	2,624

Notes: Kasparov and Karpov's games are excluded. Top 25% is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an Elo in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament, player's Elo rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.7: Performance in tournaments with and without Karpov.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
Top 25% players	-1.843	-0.205**	-0.148	-0.050	0.137*	-0.088	2.316**
	(2.357)	(0.098)	(0.399)	(0.055)	(0.071)	(0.100)	(0.895)
Mid 50% players	-0.915	-0.249***	0.091	-0.033	0.047	-0.014	1.445
	(2.103)	(0.095)	(0.289)	(0.047)	(0.064)	(0.084)	(1.042)
Bottom 25% players	2.375	-0.225*	0.820**	-0.245***	-0.066	0.311	1.445
	(2.304)	(0.135)	(0.397)	(0.076)	(0.190)	(0.245)	(1.023)
Number of moves	381,460	381,460	381,460	381,460	381,460	381,460	381,460
Number of games	5,120	5,120	5,120	5,120	5,120	5,120	5,120

Notes: Karpov's games are excluded. Top 25% is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an Elo in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament, player's Elo rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by player-year) are shown in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.8: Performance in tournaments with and without Kasparov.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
Top 25% players	-1.253*	-0.061*	-0.111	0.025	0.045	-0.069***	0.496
	(0.696)	(0.034)	(0.103)	(0.025)	(0.030)	(0.023)	(0.407)
Mid 50% players	-0.229	-0.002	-0.054	0.028*	0.003	-0.031	-0.191
	(0.538)	(0.027)	(0.072)	(0.017)	(0.023)	(0.020)	(0.387)
Bottom 25% players	0.559	-0.005	0.049	-0.027	0.012	0.014	-0.097
	(0.688)	(0.041)	(0.096)	(0.020)	(0.024)	(0.029)	(0.490)
Number of moves	420,348	420,348	420,348	420,348	420,348	420,348	420,348
Number of games	5,427	5,427	5,427	5,427	5,427	5,427	5,427

Notes: Kasparov's games are excluded. Top 25% is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an Elo in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament, player's Elo rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by player-year) are shown in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.9: Performance in tournaments with and without Carlsen.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Difficulty
Superstar effect for							
Top 25% players	-0.080	0.040	-0.128	0.010	0.022	-0.032	0.756**
	(0.698)	(0.032)	(0.097)	(0.031)	(0.038)	(0.032)	(0.362)
Mid 50% players	-1.221*	0.008	-0.186*	-0.002	0.072**	-0.069**	0.241
	(0.680)	(0.033)	(0.097)	(0.029)	(0.030)	(0.030)	(0.314)
Bottom 25% players	-0.680	-0.004	-0.073	-0.013	0.059	-0.046	0.770*
	(0.993)	(0.044)	(0.128)	(0.035)	(0.041)	(0.039)	(0.423)
Number of moves	275,260	275,260	275,260	275,260	275,260	275,260	275,260
Number of games	3,110	3,110	3,110	3,110	3,110	3,110	3,110

Notes: Carlsen's games are excluded. Top 25% is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an Elo in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament, player's Elo rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by player-year) are shown in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.10: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Carlsen Present
2019	GCT Croatia 2019, Grenke Chess Classic 2019, Gashimov Memorial 2019, Norway Chess 2019, Sinquefield 2019, Tata Steel 2019
2018	Gashimov Memorial 2018, Sinquefield 2018, Biel 2018, Norway Chess 2018, Grenke Chess Classic 2018, Tata Steel 2018
2017	London Classic 2017, Norway Chess 2017, Sinquefield 2017, Grenke Chess Classic 2017, Tata Steel 2017
2016	Norway Chess 2016, Tata Steel 2016, Bilbao Masters 2016
2015	London Classic 2015, Sinquefield 2015, Norway Chess 2015, Gashimov Memorial 2015, Grenke Chess Classic 2015, Tata Steel 2015
2014	Norway Chess 2014, Zuerich Chess Challange 2014, Sinquefield 2014, Gashimov Memorial 2014
2013	Moscow Tal Memorial 2013, Norway Chess 2013, Candidates Tournament 2013, Tata Steel 2013, Sinquefield 2013
	Panel B. Carlsen Not Present
2019	U.S. Championship 2019, Dortmund 2019
2018	Candidates Tournament 2018, U.S. Championship 2018, Dortmund 2018
2017	U.S. Championship 2017, Dortmund 2017, Gashimov Memorial 2017
2016	London Classic 2016, Sinquefield 2016, Gashimov Memorial 2016, Candidates Tournament 2016, Moscow Tal Memorial 2016, U.S. Championship 2016, Dortmund 2016
2015	Dortmund 2015, Zuerich Chess Challenge 2015, Tbilisi FIDE GP 2015, Khanty-Mansiysk FIDE GP 2015, Capablanca Memorial 2015, U.S. Championship 2015
2014	Beijing Sportaccord Basque 2014, London Classic 2014, Tashkent FIDE GP 2014, Dortmund 2014, Tata Steel 2014, U.S. Championship 2014, Candidates Tournament 2014, Baku FIDE GP 2014, Capablanca Memorial 2014, Bergomo ACP Golden Classic 2014
2013	Paris FIDE GP 2013, Dortmund 2013, Thessaloniki FIDE GP 2013, Zug FIDE GP 2013, Beijing FIDE GP 2013, Zuerich Chess Challenge 2013, Grenke Chess Classic 2013, Capablanca Memorial 2013, U.S. Championship 2013

Table A.11: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Kasparov Present
2001	Astana 2001, Zuerich 2001, Linares 2001, Corus Wijk aan Zee 2001
2000	Fujitsu Siemens Giants 2000, Sarajevo Bosnia 2000, Linares 2000, Corus Wijk aan Zee 2000
1999	Sarajevo Bosnia 1999, Linares 1999, Hoogovens Wijk aan Zee 1999
1998	Linares 1998
1997	Tilburg 1997, Novgorod 1997, Linares 1997
1996	Las Palmas 1996, Dos Hermanas 1996, Amsterdam Euwe Memorial 1996
1995	Horgen 1995, Amsterdam Euwe Memorial 1995, Novgorod 1995
	Riga Tal Memorial 1995
	Panel B. Kasparov Not Present
2001	Sigeman & Co 2001, Biel 2001, Dortmund 2001, Pamplona 2001, Dos Hermanas 2001
2000	Japfa Classic 2000, Dortmund 2000, Sigeman & Co 2000, Biel 2000
1999	Pamplona 1999, Lost Boys Amsterdam 1999, Dortmund 1999, Sigeman & Co 1999
	Dos Hermanas 1999, Biel 1999
1998	Hoogovens Wijk aan Zee 1998, Tilburg 1998, Dortmund 1998, Madrid 1998, Pamplona 1998
1997	Hoogovens Merrillville 1997, Hoogovens Wijk aan Zee 1997, Sigeman & Co 1997, Ubeda 1997,
	Dos Hermanas 1997, Lost Boys 1997, Dortmund 1997, Madrid 1997, Belgrade Investbank 1997
1996	Koop Tjuchem 1996, Donner Memorial 1996, Hoogovens Wijk aan Zee 1996,
	Tilburg 1996, Dortmund 1996, Dos Hermanas 1996, Madrid 1996
1995	Belgrade Investbank 1995, Donner Memorial 1995, Biel 1995, Madrid 1995,
	Dos Hermanas 1995, Groningen 1995, Dortmund 1995

Table A.12: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Kasparov & Karpov Both Present
1994	Linares 1994
1993	Linares 1993
1992	
1991	Reggio Emilia 1991, Tilburg 1991, Amsterdam Euwe Memorial 1991, Linares 1991
1990	
1989	World Cup Skelleftea 1989
1988	USSR Championship 1988, World Cup Belfort 1988, Optiebeurs Amsterdam 1988
1987	Brussels 1987
	Panel B. Kasparov & Karpov Neither Present
1994	Donner Memorial 1994, Dortmund 1994, Hoogovens Wijk aan Zee 1994, Groningen 1994, Munich 1994
1993	Antwerp 1993, Amsterdam VSB 1993, Madrid 1993, Las Palmas 1993, Munich 1993
1992	Alekhine Memorial 1992, Amsterdam Euwe Memorial 1992, Hoogovens Wijk aan Zee 1992, Groningen 1992, Munich 1992
1991	World Cup Reykjavik 1991, Hoogovens Wijk aan Zee 1991, Groningen 1991, Munich 1991
1990	Tilburg 1990, Hoogovens Wijk aan Zee 1990, Prague 1990, Groningen 1990, Munich 1990
1989	Hoogovens Wijk aan Zee 1989, Groningen 1989, Munich 1989, Amsterdam Euwe Memorial 1989
1988	Amsterdam Euwe Memorial 1988, OHRA Amsterdam 1988, Linares 1988, Hastings 1988
1987	Belgrade Investbanka 1987, Hoogovens Wijk aan Zee 1987, Interpolis 1987, OHRA Amsterdam 1987, Reykjavik 1987

Table A.13: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Karpov Present
1983	Interpolis 1983, International DSB Mephisto GM 1983, USSR Final 1983, Bath 1983, Linares 1983
1982	Interpolis 1982, Turin 1982, Hamburg 1982, London Phillips 1982, Mar del Plata Clarin Masters 1982
1981	IBM Herinnerungs Toernooi 1981, Moscow 1981, Linares 1981
1980	Buenos Aires 1980, Interpolis 1980, IBM Kroongroep 1980, Bugojno 1980, Bad Kissingen 1980
1979	Interpolis 1979, Waddinxveen KATS 1979, Montreal International 1979, GER International 1979
1978	Bugojno 1978
1977	Interpolis 1977, October Revolution 1977, Las Palmas 1977, GER International 1977
1976	USSR Final 1976, Montilla 1976, Manila Marlboro 1976, Amsterdam 1976, Skopje Solidarnost 1976
	Panel B. Karpov Not Present
1983	Jakarta International 1983, Hoogovens Wijk aan Zee 1983
1982	Bugojno 1982, Moscow Interzonal 1982, Las Palmas Interzonal 1982, Toluca Interzonal 1982, Niksic International 1982, Hoogovens Wijk aan Zee 1982
1981	Las Palmas 1981, IBM Herinnerungs Toernooi 1981, Interpolis 1981, Hoogovens Wijk aan Zee 1981
1980	Buenos Aires 1980, London Phillips 1980, Hoogovens Wijk aan Zee 1980, Las Palmas 1980, Reykjavik International 1980
1979	Buenos Aires Clarin 1979, Riga Interzonal 1979, Buenos Aires Interzonal 1979, Vidmar Memorial 1979, IBM 1979, Hoogovens Wijk aan Zee 1979, Buenos Aires Konex 1979
1978	Interpolis 1978, Reykjavik International 1978, Hoogovens Wijk aan Zee 1978, Las Palmas 1978 IBM 1978, Clarin 1978
1977	Geneve 1977, Vidmar Memorial 1977, Hoogovens Wijk aan Zee 1977, IBM 1977
1976	Interzonal 1976, Las Palmas 1976, Reykjavik International 1976, Hoogovens Wijk aan Zee 1976, IBM 1976

Table A.14: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Fischer Present
1970	Interzonal 1970, Buenos Aires 1970, Rovinj Zagreb 1970
1969	
1968	Vinkovci 1968, Nathanya 1968,
1967	Skopje 1967, Monaco Grand Prix 1967
1966	Piatigorsky Cup 1966, U.S. Championship 1966
1965	U.S. Championship 1965, Capablanca Memorial 1965
1964	
1963	U.S. Championship 1963
1962	U.S. Championship 1962, Candidates Tournament 1962, Interzonal 1962
	Panel B. Fischer Not Present
1970	Vinkovci 1970, IBM Amsterdam 1970, Budapest 1970, Sarajevo 1970, Caracas 1970, Hoogovens Wijk an Zee 1970, Costa del Sol 1970, Skopje 1970, Rubinstein Memorial 1970, Christmas Congress 1970
1969	Monaco Grand Prix 1969, Hoogovens Wijk an Zee 1969, Venice 1969 U.S. Championship 1969, Palma de Mallorca 1969, IBM Amsterdam 1969, Sarajevo 1969, Christmas Congress 1969, Rubinstein Memorial 1969, Capablanca Memorial 1969
1968	Rubinstein Memorial 1968, Christmas Congress 1968, Palma de Mallorca 1968, U.S. Championship 1968, Bamberg 1968, IBM Amsterdam 1968, Sarajevo 1968 Hoogovens Wijk an Zee 1968, Monaco Grand Prix 1968, Skopje 1968
1967	Winnipeg 1967, October Revolution Leningrad 1967, October Revolution Moscow 1967, Capablanca Memorial 1967, Palma de Mallorca 1967, Sarajevo 1967, Hoogovens Beverwijk 1967, Christmas Congress 1967, Rubinstein Memorial 1967, Venice 1967, IBM Amsterdam 1967
1966	IBM Amsterdam 1966, Sarajevo 1966, Palma de Mallorca 1966 Hoogovens Beverwijk 1966, Venice 1966, Rubinstein Memorial 1966, Christmas Congress 1966
1965	ZSK International 1965, Zagreb 1965, Mer del Plata 1965, IBM Amsterdam 1965, Sarajevo 1965, Hoogovens Beverwijk 1965, Christmas Congress 1965, Rubinstein Memorial 1965
1964	Buenos Aires 1964, Capablanca Memorial 1964, Rubinstein Memorial 1964, Interzonal 1964, IBM Amsterdam 1964, Sarajevo 1964, Hoogovens Beverwijk 1964, Christmas Congress 1964, ZSK International 1964
1963	Piatigorsky Cup 1963, Alekhine Memorial 1963, IBM Amsterdam 1963, Sarajevo 1963, Hoogovens Beverwijk 1963, Rubinstein Memorial 1963, Christmas Congress 1963, Capablanca Memorial 1963
1962	Mer del Plata 1962, Sarajevo 1962, Hoogovens Beverwijk 1962, Rubinstein Memorial 1962, Christmas Congress 1962, Capablanca Memorial 1962

Table A.15: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Hou Yifan Present
2015	Monte Carlo FIDE GP 2015
2014	Lopota FIDE GP 2014, Khanty-Mansiysk FIDE GP 2014, Sharjah FIDE GP 2014
	Panel B. Hou Yifan Not Present
2019	Skolkovo FIDE GP 2019, Saint Louis Cairns Cup 2019
2016	Khanty-Mansiysk FIDE GP 2016, Chengdu FIDE GP 2016, Batumi FIDE GP 2016, Tehran FIDE GP 2016